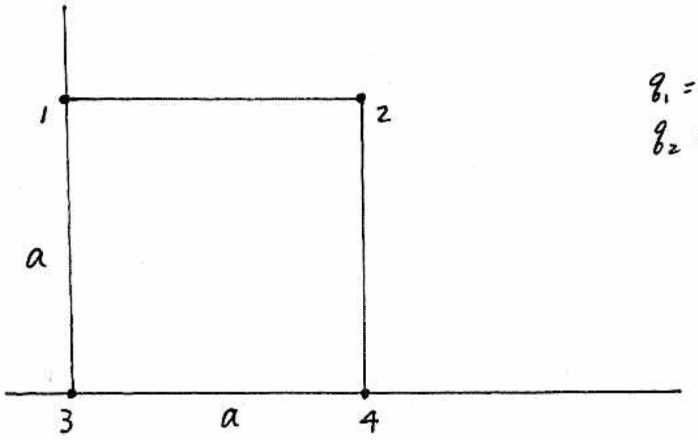
Ch 21 . # 6, 20, 25, 65. 66

#6.



$q_1 = q_4 = Q$
$q_2 = q_3 = q$

(a) $\Sigma \vec{F}_{on1} = \vec{F}_{2on1} + \vec{F}_{3on1} + \vec{F}_{4on1} = 0$ (Given condition)

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{\imath} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{\jmath} + \frac{1}{4\pi\varepsilon}\frac{Q^2}{(\sqrt{2}a)^2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$$

$$= \frac{1}{4\pi\varepsilon}\frac{Q}{a^2}\left(q + \frac{Q}{2\sqrt{2}}\right)\hat{\imath} + \frac{1}{4\pi\varepsilon}\frac{Q}{a^2}\left(q + \frac{Q}{2\sqrt{2}}\right)\hat{\jmath} = 0\hat{\imath} + 0\hat{\jmath}$$

$$\therefore \quad q + \frac{Q}{2\sqrt{2}} = 0$$

$$\therefore \quad \frac{Q}{q} = -2\sqrt{2} = \underline{\underline{-2.828427125}}$$ (they are opposite charges)

$\Sigma \vec{F}_{on3} = \vec{F}_{1on3} + \vec{F}_{2on3} + \vec{F}_{4on3} = 0$

$$= \frac{1}{4\pi\varepsilon}\frac{Qq}{a^2}\hat{\jmath} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\sqrt{2}a)^2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{\imath} = 0\hat{\imath} + 0\hat{\jmath}$$

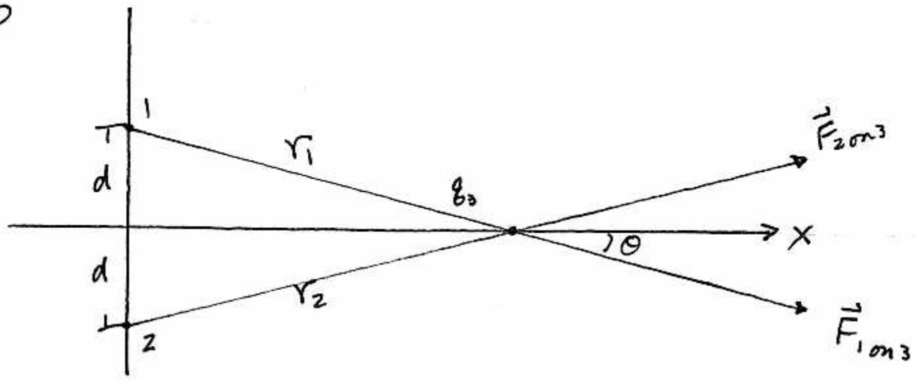
$$= \frac{1}{4\pi\varepsilon}\frac{q}{a^2}\left(Q + \frac{q}{2\sqrt{2}}\right)\hat{\imath} + \frac{1}{4\pi\varepsilon}\frac{q}{a^2}\left(Q + \frac{q}{2\sqrt{2}}\right)\hat{\jmath}$$

$$\therefore \quad Q + \frac{q}{2\sqrt{2}} = 0$$

$$\therefore \quad \frac{Q}{q} = -\frac{1}{2\sqrt{2}} = \underline{\underline{-0.35355339}}$$

(b) There is no way to satisfy both conditions at the same time.

#20



$q_1 = q_2 = 3.20 \times 10^{-19} C$

$q_3 = 6.40 \times 10^{-19} C \; (= 2q_1)$

$d = 0.17 m$

$0 cm \leq x \leq 5 m$

DO NOT PlUG IN NUMBERS UNTIL THE END !

Because $q_1 = q_2$ and $r_1 = r_2$, $|F_{1on3}| = |F_{2on3}|$.

Also, because of the symmetry $\Sigma \vec{F}_y = 0$. therefore $\Sigma \vec{F}_x = 2 |F_{1on3,x}|$

$$\Sigma \vec{F}_x = 2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_1^2} \cos\theta \qquad \cos\theta = \frac{x}{r_1} \;\&\; r_1 = (d^2+x^2)^{1/2}$$

$$= 2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{q_1(=2q_1)}{r_1^2} \cdot \frac{x}{r_1}$$

$$= \frac{4 q_1^2}{4\pi\varepsilon_0} \cdot \frac{x}{r_1^3} = \underline{\frac{q_1^2}{\pi\varepsilon_0} \frac{x}{(d^2+x^2)^{3/2}}} \text{ ———————— } ①$$

this is a typical maximization / minimization problem.

$$\frac{d\vec{F}_x}{dx} = \frac{d\left[\frac{q_1^2}{\pi\varepsilon_0} \frac{x}{(d^2+x^2)^{3/2}}\right]}{dx} = \frac{q_1^2}{\pi\varepsilon_0} \left(\frac{1}{(d^2+x^2)^{3/2}} - \frac{3}{2} x (d^2+x^2)^{-5/2} \cdot 2x \right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0} \left(\frac{1}{(d^2+x^2)^{3/2}} - \frac{3x^2}{(d^2+x^2)^{5/2}} \right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0} \left(\frac{d^2+x^2-3x^2}{(d^2+x^2)^{5/2}} \right)$$

$$= \frac{q_1^2}{\pi\varepsilon_0} \left(\frac{d^2-2x^2}{(d^2+x^2)^{5/2}} \right) = 0$$

$$\therefore \; d^2 - 2x^2 = 0$$

$$\therefore \; \underline{\underline{x = \frac{d}{\sqrt{2}}}}$$

(a) & (c) Minimum is at $\underline{X = 0}$ (Forces from q_1 & q_2 are equal & opposite $\therefore \underline{\Sigma \vec{F} = 0}$)

(b) & (d) Max. is at $X = \frac{d}{\sqrt{2}} = \underline{\underline{0.120208152\ m}}$ ——— ②

①←②

$$\Sigma \vec{F}_x = \frac{q_1^2}{\pi \varepsilon_0} \frac{x}{(d^2 + x^2)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \cdot \frac{d/\sqrt{2}}{\left(d^2 + \left(\frac{d}{\sqrt{2}}\right)^2\right)^{3/2}}$$

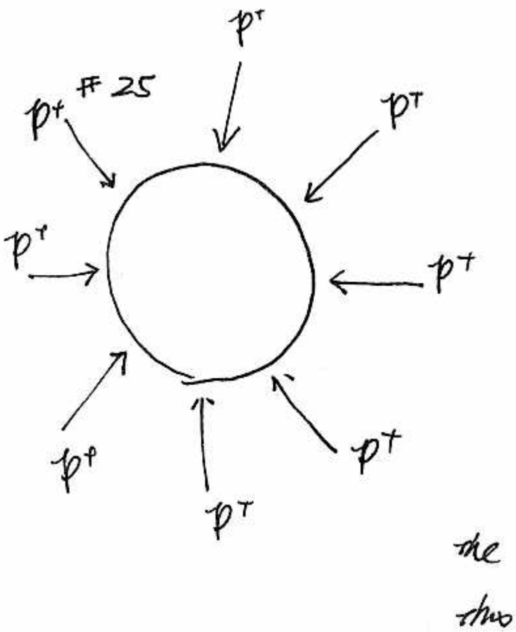
$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\left(d^2 + \frac{d^2}{2}\right)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\left(\frac{3d^2}{2}\right)^{3/2}}$$

$$= \frac{q_1^2}{\pi \varepsilon_0} \frac{d/\sqrt{2}}{\frac{3^{3/2}}{2^{3/2}} \cdot d^3} = \frac{q_1^2}{\pi \varepsilon_0} \cdot \frac{2}{3^{3/2} \cdot d^2}$$

$$= \frac{(3.20 \times 10^{-19} C)^2}{\pi \cdot 8.85 \times 10^{-12}} \cdot \frac{2}{\sqrt{27} \cdot (0.17)^2} = \underline{\underline{4.90520417 \times 10^{-26}\ N}}$$

#25



rate : $1500\ p^+/m^2/sec$

Total surface area of the earth $= 4\pi R_\oplus^2$

$R_\oplus = 6.37 \times 10^6 m$

Hence the total # of protons hitting the earth :

$$1500\ p^+/m^2/sec \cdot 4\pi R_\oplus^2 = 7.648565457 \times 10^{17}\ p^+/sec$$

$$= \frac{dq}{dt}$$

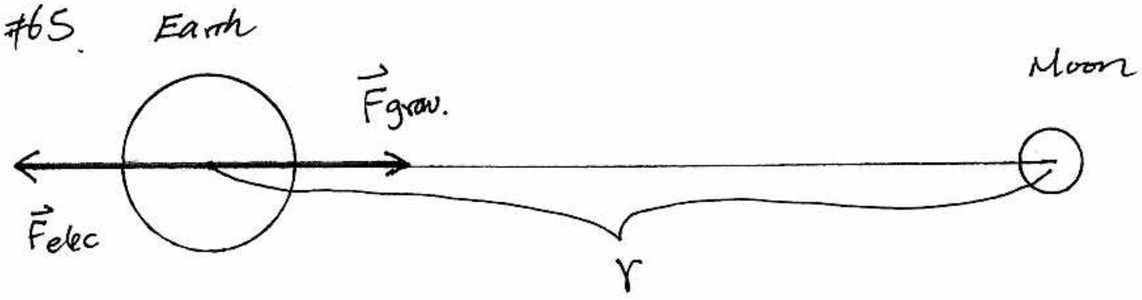
the earth is getting 7.65×10^{17} protons per sec. means this is the rate of change of charge ($\frac{dq}{dt}$) which is "current". We have to change the charge into coulombs

$$7.648565457 \times 10^{17}\ p^+/sec \times \frac{1.6 \times 10^{-19} C}{1\ p^+}$$

$$= 1.22 \times 10^{-1}\ c/sec$$

$$= \underline{\underline{0.122\ amp}}$$

#65.



$$F_{grav.} = G\frac{m_\oplus m_☾}{r^2}$$

$$F_{elec} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad (q_1 = q_2 \;:\; \text{Given condition})$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1^2}{r^2}$$

(a)

$$\Sigma\vec{F} = G\frac{M_\oplus M_☾}{r^2} - \frac{1}{4\pi\varepsilon}\frac{q_1^2}{r^2} = 0$$

Solve for q_1

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1^2}{r^2} = G\frac{M_\oplus M_☾}{r^2}$$

$$\therefore\; q_1 = \sqrt{4\pi\varepsilon_0 G M_\oplus M_☾}$$

$$= \sqrt{4\cdot\pi\cdot 8.85\times10^{-12}\cdot 6.6726\times10^{-11}\cdot 5.98\times10^{24}_{kg}\cdot 7.36\times10^{22}_{kg}}$$

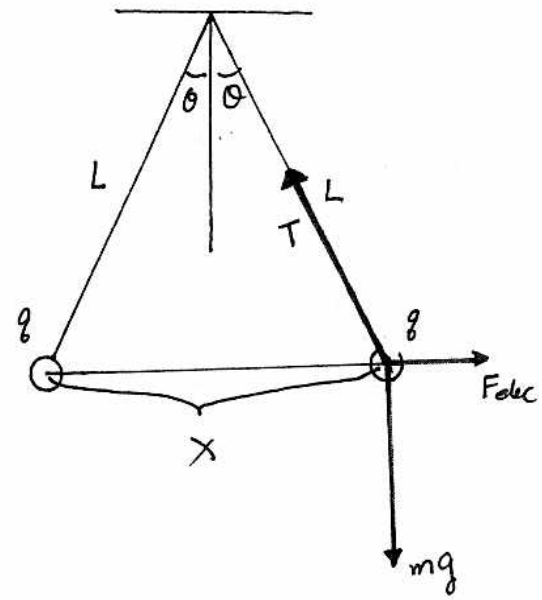
$$= \underline{\underline{5.714965488\times10^{13}\,C}}$$

(b) Both ($F_{grav.}$ & $F_{elec.}$) are ruled by Inverse-Square-Law so when they are set equal to each other, r^2 is cancelled.

(c) $1\,p^+ = 1.6\times10^{-19}C\;,\quad 1.673\times10^{-27}kg$

$$5.7149\cdots\times10^{13}C\cdot\frac{1.673\times10^{-27}kg/\text{proton}}{1.6\times10^{-19}C/\text{proton}} = \underline{\underline{5.975710788\times10^{5}\,kg}}$$

#66. Just because you are in Phy. 231 does not mean you can forget what you learned in Phy. 230. You will encounter lots of problems that require the methods you learned in 230. This is one of them.



Steps (Just in case of you forgot)

1. Draw a diagram
2. Pick a point
3. Draw a vector / vectors applied to the point
4. Break it / them into x - y components.
5. Write eqns (for both x & y) about the point

(a) In this case, there are three forces (see the diagram) and $\Sigma \vec{F} = 0$

X - comp

$F_{elec} - T\sin\theta = 0$

$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} - T\sin\theta = 0$

$\therefore T\sin\theta = \frac{1}{4\pi\varepsilon}\frac{q^2}{X^2}$ —— ①

y - comp

$T\cos\theta - mg = 0$

$\therefore T\cos\theta = mg$ —— ②

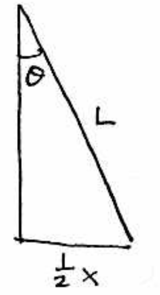
$\frac{①}{②}$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{X^2}}{mg}$$

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{X^2 mg}$$

$\wr$

$$\sin\theta \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{X^2 mg}$$



$\sin\theta = \frac{\frac{1}{2}X}{L}$

$$\frac{\frac{1}{2}X}{L} = \frac{1}{4\pi\varepsilon}\frac{q^2}{X^2 mg}$$

$$X^3 = \frac{1}{4\pi\varepsilon}\frac{2Lq^2}{mg}$$

$$\therefore X = \left(\frac{L}{2\pi\varepsilon_0}\frac{q^2}{mg}\right)^{1/3}$$

(b) Solve for q : $q = \pm\sqrt{\frac{X^3 \cdot 2\pi\varepsilon_0 m g}{L}}$

$X = 5cm = 5\times10^{-2} m$
$L = 120 cm = 1.2 m$
$m = 10g = 1\times10^{-2} kg$

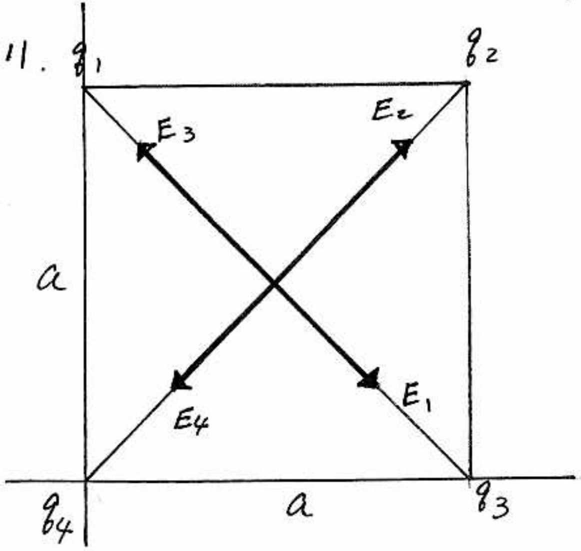
Do Not Forget to convert them into SI units!!

$$q = \pm\sqrt{\frac{(5\times10^{-2})^3 \cdot 2 \cdot \pi\,\varepsilon_0\,(1\times10^{-2})(9.81)}{1.2}}$$

$$= \underline{\underline{\pm\ 2.384\times10^{-8} C}}$$

Ch.22 # 11, 27, 29, 55, 84, 86

\# 11.



$a = 5\,cm$

$q_1 = +10\,nC$

$q_2 = -20\,nC$

$q_3 = 20\,nC$

$q_4 = -10\,nC$

(As you can see $\Sigma \vec{E}_x = 0$ ·· only noticable when you draw a diagram very accurate)

$\therefore$ $\vec{E}_{Total}$ is up.

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon}\left(\frac{q_1}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} - \frac{q_1}{\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

[treating all the charges as positive since we know their directions]

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} + \frac{q_2}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon}\left(-\frac{q_3}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} + \frac{q_3}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\vec{E}_4 = \frac{1}{4\pi\varepsilon}\left(-\frac{q_4}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{i} - \frac{q_4}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\Sigma\vec{E} = \frac{1}{4\pi\varepsilon}\left[\left(\frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\frac{1}{\sqrt{2}}\right)\hat{i} + \left(-\frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{10nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(0\hat{i} + \frac{20nC}{(\frac{\sqrt{2}}{2}a)^2}\cdot\frac{1}{\sqrt{2}}\hat{j}\right)$$

$$= \underline{\underline{0\hat{i} + 1.017306572\times10^{5}\,N/C\ \hat{j}}}$$

\# 27.

(a) $\lambda = \frac{Q}{L} = \frac{-4.23\times10^{-15}C}{0.0815m} = -5.190184049\times10^{-14}\,C/m$

For the calculations of E fields for non-point charged objects, use the same steps. Do Not forget these steps practice!

Step 1

Draw a diagram (of course). Take a small piece of a charged object (dq). Draw an E field line from it at the point.

Step 2

Write an eqn. of dE created by dq at the point.

Step 3

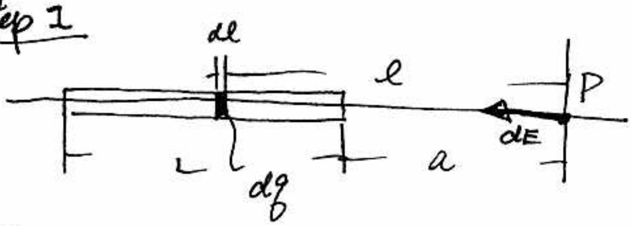
check symmetry to see if Ex or Ey is cancelled.

Step 4

Integrate.

(b)

Step 1



dE is toward the rod because the charge is negative.

Step 2

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{l^2} \qquad dq = \lambda \cdot dl$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot dl}{l^2}$$

Step 3

In this case, there is no "y" direction.

Step 4

$$E = \int dE = \int_a^{a+L} \frac{1}{4\pi\varepsilon_0}\frac{\lambda \cdot dl}{l^2}$$

$$= \frac{\lambda}{4\pi\varepsilon} \left. -\frac{1}{l} \right|_a^{a+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon}\left(\frac{L}{a(a+L)}\right)$$

(= $1.573008017 \times 10^{-3}$ N/C w/ values given)

(c) As noted in (b), the direction of the E field is toward the rod

(d) w/ $a = 50m$ $E = 1.518937486 \times 10^{-8}$ N/C

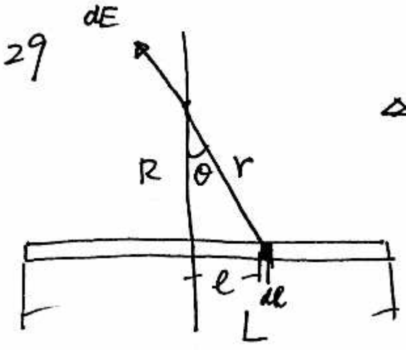
(e) $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 1.521413354 \times 10^{-8}$ N/C

The questions are trying to tell you that when the solution is

$\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{a(a+L)}\right)$ and $a >>>>> L$, the result is $\sim \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{a^2}$

(remember $\lambda = \frac{q}{L}$ and $\lambda L = q$. so you can treat as a point charge)

\# 29



Step 1

Step 2

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cdot dl}{(l^2+R^2)} \quad \begin{cases} \lambda = \frac{Q}{L} \\ r = (R^2+l^2)^{1/2} \end{cases}$$

Step 3

$E_x = 0$ by symmetry (Don't forget to write this even if this is obvious to you!)

Step 4

$$dE_y = dE \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda dl}{(l^2+R^2)} \cdot \frac{R}{(R^2+l^2)^{1/2}} \qquad \left(\cos\theta = \frac{R}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda R \, dl}{(l^2+R^2)^{3/2}}$$

since there is no "l" in the numerator → Trig sub

Let $\tan\theta = \frac{l}{R}$

$$\begin{cases} l = R\tan\theta \\ dl = R\sec^2\theta \cdot d\theta \end{cases}$$

So

$$dE_y = \frac{\lambda}{4\pi\varepsilon_0} \frac{R\,(R\sec^2\theta \cdot d\theta)}{(R^2\tan^2\theta + R^2)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \frac{R^2\sec^2\theta \cdot d\theta}{R^3(\tan^2\theta+1)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{R} \frac{\sec^2\theta \cdot d\theta}{\sec^3\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cdot \frac{1}{\sec\theta} \cdot d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cos\theta \cdot d\theta$$

$$E_y = \int dE_y = \int_{\theta_{min}}^{\theta_{max}} \frac{\lambda}{4\pi\varepsilon R} \cos\theta \cdot d\theta$$

$$= 2\int_0^{\theta_{max}} \frac{\lambda}{4\pi\varepsilon R} \cos\theta \cdot d\theta$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \sin\theta \Big|_0^{\theta_{max}}$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \frac{l}{(R^2+l^2)^{1/2}} \Big|_0^{l=\frac{1}{2}L}$$

$$= \frac{2\lambda}{4\pi\varepsilon R} \left(\frac{\frac{1}{2}L}{(R^2+(\frac{1}{2}L)^2)^{1/2}} \right) = \frac{\lambda}{2\pi\varepsilon_\circ R} \frac{\frac{1}{2}L}{(R^2+\frac{1}{4}L^2)^{1/2}}$$

$$= \frac{\lambda}{2\pi\varepsilon R} \cdot \frac{\frac{1}{2}L}{\frac{1}{2}(4R^2+L^2)^{1/2}} = \underline{\underline{\frac{\lambda}{2\pi\varepsilon_0 R} \frac{L}{(4R^2+L^2)^{1/2}}}}$$

$$= \frac{\frac{Q}{L}}{2\pi\varepsilon R} \frac{L}{(4R^2+L^2)^{1/2}}$$

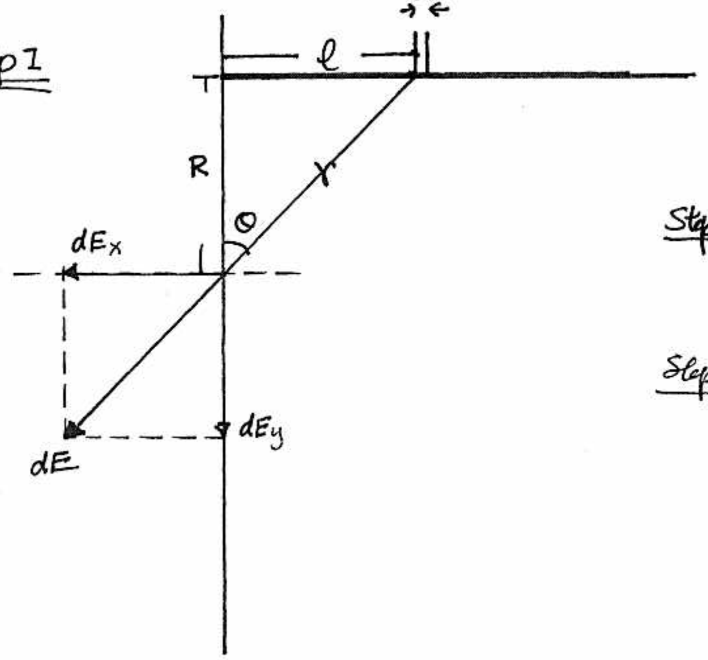
$$= \underline{\underline{\frac{Q}{2\pi\varepsilon R} \frac{1}{(4R^2+L^2)^{1/2}}}}$$

(a) $E_y = \dfrac{7.81 \times 10^{-9}}{2\pi\varepsilon_0 (0.145m)} \cdot \dfrac{1}{(4(0.06)^2+(0.145)^2)^{1/2}} = \underline{\underline{2.734324373 \times 10^4 \, N/C}}$

(b) <u>+ y direction</u>

#55

Step I



Step 2

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2}$$

Step 3

In this case there's no symmetry ☹

X comp

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \sin\theta$$ (Not worrying about the direction)

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dl}{(l^2+R^2)} \cdot \frac{l}{(l^2+R^2)^{1/2}}$$

y comp

$$dE_y = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dl}{(l^2+R^2)} \cdot \frac{R}{(l^2+R^2)^{1/2}}$$

Step 4

$$E_x = \int dE_x = \int \frac{\lambda}{4\pi\varepsilon} \frac{l\, dl}{(l^2+R^2)^{3/2}}$$ (u-sub)

Let $\begin{cases} u = l^2+R^2 \\ du = 2l\, dl \end{cases}$

$$= \int \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{2} \frac{2l\, dl}{(l^2+R^2)^{3/2}}$$

$$= \frac{\lambda}{8\pi\varepsilon} \int \frac{du}{u^{3/2}}$$

$$= \frac{\lambda}{8\pi\varepsilon_0} \; -2\frac{1}{u^{1/2}}$$

$$= -\frac{\lambda}{4\pi\varepsilon} \cdot \frac{1}{u^{1/2}}$$

$$= \frac{-\lambda}{4\pi\varepsilon} \frac{1}{(l^2+R^2)^{1/2}} \Big|_0^{\infty}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{R}$$

$$E_y = \int dE_y = \int \frac{\lambda R}{4\pi\varepsilon_0} \frac{dl}{(l^2+R^2)^{3/2}}$$ (Trig-sub)

Let $\tan\theta = \frac{l}{R}$

$\begin{cases} l = R\tan\theta \\ dl = R\sec^2\theta \cdot d\theta \end{cases}$

$$= \frac{\lambda R}{4\pi\varepsilon} \int \frac{R\sec^2\theta \cdot d\theta}{(R^2\tan^2\theta + R^2)^{3/2}}$$

$$= \frac{\lambda R^2}{4\pi\varepsilon_0} \int \frac{\sec^2\theta \cdot d\theta}{R^3(\tan^2\theta + 1)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R} \cdot \int \frac{\sec^2\theta \cdot d\theta}{\sec^3\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon R} \int \frac{1}{\sec\theta} \cdot d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon R} \int \cos\theta \cdot d\theta$$

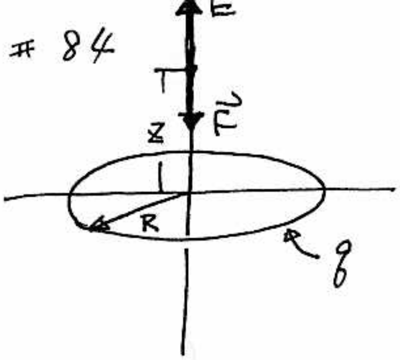
$$= \frac{\lambda}{4\pi\varepsilon_0 R} \sin\theta \Big|_0^{\pi/2}$$

$$= \frac{\lambda}{4\pi\varepsilon R}$$

$$\theta_{final} = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1)$$

$$= \underline{\underline{45^\circ}}$$

84



$$E = \frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$

$$F = -Ee^- = m_e a_z$$

Because $\vec{F}$ & $\vec{E}$ are opposite to each other

$$\therefore -\frac{qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot e^- = m_e a_z$$

$$-\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot z = m_e \ddot{z} \qquad \left(\ddot{z} = \frac{d^2 z}{dt^2} = a_z \right)$$

$$\frac{qe^-}{4\pi\varepsilon (z^2+R^2)^{3/2}} z + m_e \ddot{z} = 0$$ — you should be able to recognize this is a siple harmonic oscillation!

Let $z = A\cos(\omega t + \phi)$

$\dot{z} = -A\omega \sin(\omega t + \phi)$

$\ddot{z} = -A\omega^2 \cos(\omega t + \phi)$

$$\frac{qe^-}{4\pi\varepsilon (z^2+R^2)^{3/2}} A\cos(\omega t+\phi) - m_e A\omega^2 \cos(\omega t+\phi) = 0$$

$$\left(\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} - m_e \omega^2 \right)\left(A\cos(\omega t+\phi) \right) = 0$$

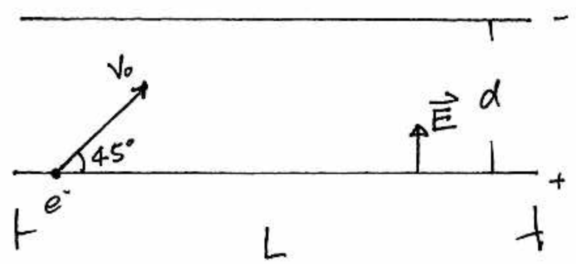
$$\therefore \frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} - m_e \omega^2 = 0$$

$$\therefore \quad \omega = \sqrt{\frac{qe^-}{4\pi\varepsilon_0 (z^2+R^2)^{3/2} m_e}}$$

for a small displacement ($R >> z$)

$$\omega = \sqrt{\frac{qe^-}{4\pi\varepsilon_0 m_e R^3}}$$

\# 86



$V_0 = 6 \times 10^6$ m/sec

$E = 2 \times 10^3$ N/C

$d = 2$ cm $= 0.02$ m

$L = 10$ cm $= 0.1$ m

X comp

$a_x = 0$ ——— ①

$V_x = V_0 \cos 45^\circ$ ——— ②

$X = V_0 t \cos 45^\circ \; (\int V_x \cdot dt)$ —— ③

y comp

$F_{elec} = -Ee^-$ (F on the electron is down)

We can ignore $F_{grav.}$ because $F_{elec} >> F_{grav.}$

$F_{elec} = -Ee^- = m_e a_y$ ——— ④

$\therefore \; a_y = -\frac{Ee^-}{m_e}$

$V_y = \int a_y \cdot dt = -\frac{Ee^-}{m_e}t + \overset{V_0 \sin 45^\circ}{\cancel{V_{0y}}}$ —— ⑤

$y = \int V_y \cdot dt = -\frac{Ee^-}{2m_e}t^2 + V_0 t \sin 45^\circ + \overset{0}{\cancel{y_0}}$ —⑥

Don't be scare by all these letters.
this is another application of Physics 230. We are using; F = ma, a, V, & X or y relations. In this case, the applied force is electrical, not gravitational - that is the only difference.

(a) Let's check to see if the e^- will hit the top plate
Eqn. ⑥. set $y = 2 \times 10^{-2}$ m, solve for t

$$2 \times 10^{-2} = -\frac{Ee^-}{2m_e}t^2 + V_0 t \sin 45^\circ$$

$$\frac{Ee^-}{2m_e}t^2 - V_0 t \overset{\frac{1}{\sqrt{2}}}{\cancel{\sin 45^\circ}} + 2 \times 10^{-2} = 0$$

$$t = \frac{\frac{V_0}{\sqrt{2}} \pm \sqrt{\left(\frac{V_0}{\sqrt{2}}\right)^2 - 4\left(\frac{Ee^-}{2m_e} \cdot 2 \times 10^{-2}\right)}}{2 \cdot \frac{Ee^-}{2m_e}}$$

$$= \frac{4.24 \times 10^{-6} \pm 1.9873 \times 10^{6}}{3.5126 \times 10^{14}}$$

$$= 6.421 \times 10^{-9} \text{ sec or } 1.77 \times 10^{-8} \text{ sec}$$

the second solution is as if there is no top plate & the E field is constant (above the space of the top plate)

In reality, the electron will hit the top plate at 6.421×10^{-9} sec

→ Yes, it does hit the top plate (at $t = 6.421 \times 10^{-9}$ sec — ⑥′)

(b) ③ ← ⑥′

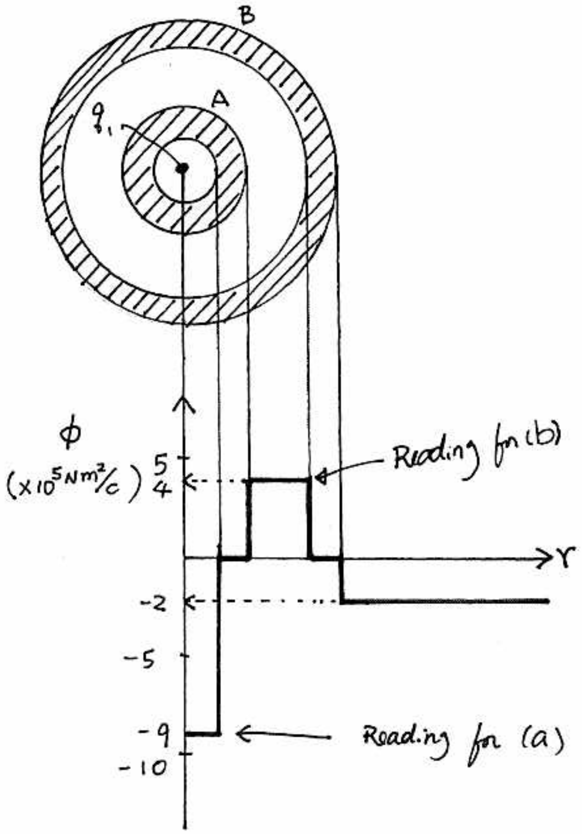
$$X = V_0 t \cos 45°$$

$$= 6. \times 10^{6} . \ t \cdot \frac{1}{\sqrt{2}}$$

$$\underline{\underline{= 2.724 \times 10^{-2} m}}$$

Ch 23. #18, 26, 29, 30, 39, 47, 48, 50, 53 √extra
83

#18



(a) $\Phi_E = \oint E\cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$= E\cdot 4\pi r^2 = \frac{q_1}{\varepsilon_0} = -9\times10^5\ Nm^2/C$

$\therefore q_1 = \varepsilon_0(-9\times10^5\ Nm^2/C) = \underline{\underline{-7.965\times10^{-6}\ C}}$

(b) $\Phi_E = \oint E\cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$= E\cdot 4\pi r^2 = \frac{q_1+q_A}{\varepsilon_0} = 4\times10^5\ Nm^2/C$

$\therefore q_A = \varepsilon_0(4\times10^5\ Nm^2/C) - q_1$

$= \varepsilon_0(4\times10^5\ Nm^2/C) - \varepsilon_0(-9\times10^5\ Nm^2/C)$

$= \varepsilon_0(13\times10^5\ Nm^2/C) = \underline{\underline{1.1505\times10^{-5}\ C}}$

(c) $\Phi_E = \oint E\cdot dA = \frac{q_{enc}}{\varepsilon_0}$

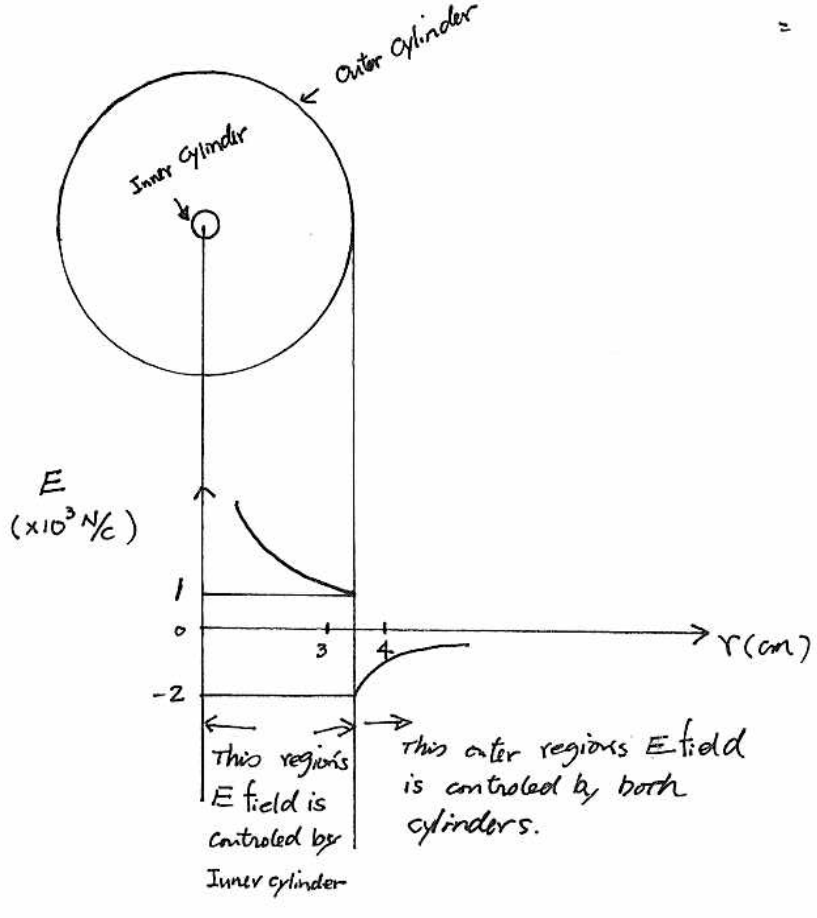
$= E\cdot 4\pi r^2 = \frac{q_1+q_A+q_B}{\varepsilon_0} = -2\times10^5\ Nm^2/C$

$q_B = \varepsilon_0(-2\times10^5\ Nm^2/C) - (q_1+q_A)$

$= \varepsilon_0(-2\times10^5\ Nm^2/C) - \varepsilon_0(4\times10^5\ Nm^2/C)$

$= \varepsilon_0(-6\times10^5\ Nm^2/C) = \underline{\underline{-5.31\times10^{6}\ C}}$

#28



$R_{\text{inner cylinder}} < r < R_{\text{outer cylinder}}$

$$\oint E_1\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_1\ 2\pi r L = \frac{\lambda_{\text{inner}} \cdot L}{\varepsilon_0}$$

$$\lambda_{\text{inner}} = E_1\,(2\pi r)\,\varepsilon_0 \qquad (E_1 = 1\times10^{3}\,\text{N/C} \text{ at } 3.5\,\text{cm } (0.035\,\text{m})) \text{ — ①}$$

$R_{\text{outer cylinder}} < r$

$$\oint E_2\, dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_2 \cdot 2\pi r L = \frac{(\lambda_{\text{inner}} + \lambda_{\text{outer}})L}{\varepsilon_0}$$

$$\lambda_{\text{outer}} = E_2\,(2\pi r)\,\varepsilon_0 - \lambda_{\text{inner}} \qquad (E_2 = -2\times10^{3}\,\text{N/C at } 0.035\,\text{m})$$

Sub. eqn ①

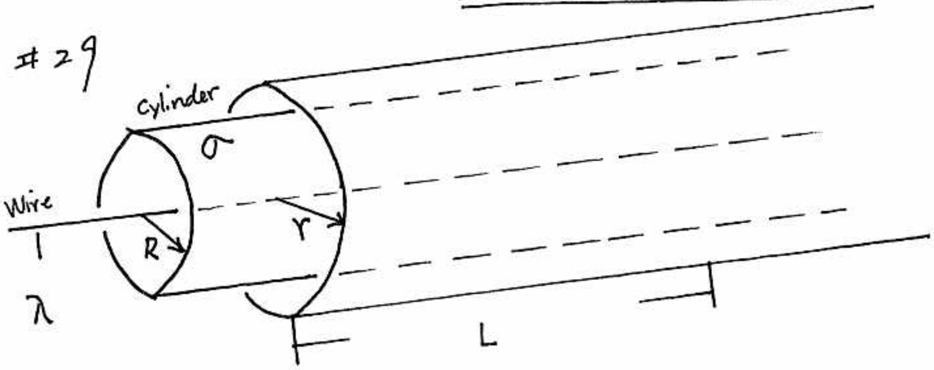
$$= E_2\,(2\pi r)\,\varepsilon_0 - E_1\,(2\pi r)\,\varepsilon_0$$

$$= (E_2 - E_1)(2\pi r)\,\varepsilon_0$$

$$= (-2\times10^{3}\,\text{N/C} - 1\times10^{3}\,\text{N/C})(2\pi\,(0.035\,\text{m}))\ 8.85\times10^{-12}$$

$$= \underline{\underline{-5.838649947\times10^{-9}\,\text{C/m}}}$$

#29



For $r > R$, $E = 0$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

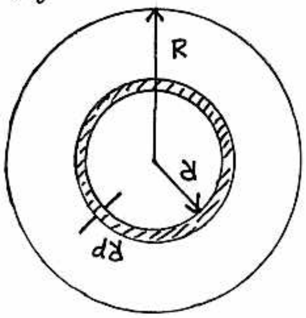
$$E \cdot 2\pi r \cdot L = \frac{\lambda L + \sigma(2\pi R)L}{\varepsilon_0}$$

$$E = \frac{\lambda + \sigma\,(2\pi R)}{2\pi r\,\varepsilon_0} = 0$$

$$\therefore\ \lambda + \sigma\,(2\pi R) = 0$$

$$\sigma = \frac{-\lambda}{2\pi R} = \frac{-(-3.6\times10^{-9}\,\text{C/m})}{2\pi\ (1.5\times10^{-2}\,\text{m})} = \underline{\underline{3.8197\times10^{-8}\,\text{C/m}^2}}$$

38



$$dq = \rho\,(2\pi\gamma \cdot d\gamma)\,L \quad (L \text{ for cylinder})$$
$$= A\gamma^2\,(2\pi\gamma\, d\gamma)\,L$$
$$= A\; 2\pi L\; \gamma^3 \cdot d\gamma$$

$$\therefore\; q \;(\text{from the center to } r < R) = \int dq$$
$$= \int_0^r A\, 2\pi L\, \gamma^3 \cdot d\gamma$$
$$= A\, 2\pi L\, \tfrac{1}{4}\, \gamma^4 \Big|_0^r$$
$$= \frac{A\pi L r^4}{2}$$

(a) for $r < R$ $\quad$ ($r = 0.03\,m$, $R = 0.04\,m$ in this case)

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E\, 2\pi r L = \frac{\int dq}{\varepsilon_0} \longrightarrow \text{see above for evaluation of } q$$
$$E\, 2\pi r L = \frac{A\pi L r^4}{2\varepsilon_0}$$
$$\therefore\; E = \frac{Ar^3}{4\varepsilon_0} = \frac{2.5\times 10^{-6}\,\text{c/m}^2 \cdot (0.03)^3}{4\;\;\varepsilon_0} = \underline{\underline{1.906779661\ \text{N/C}}}$$

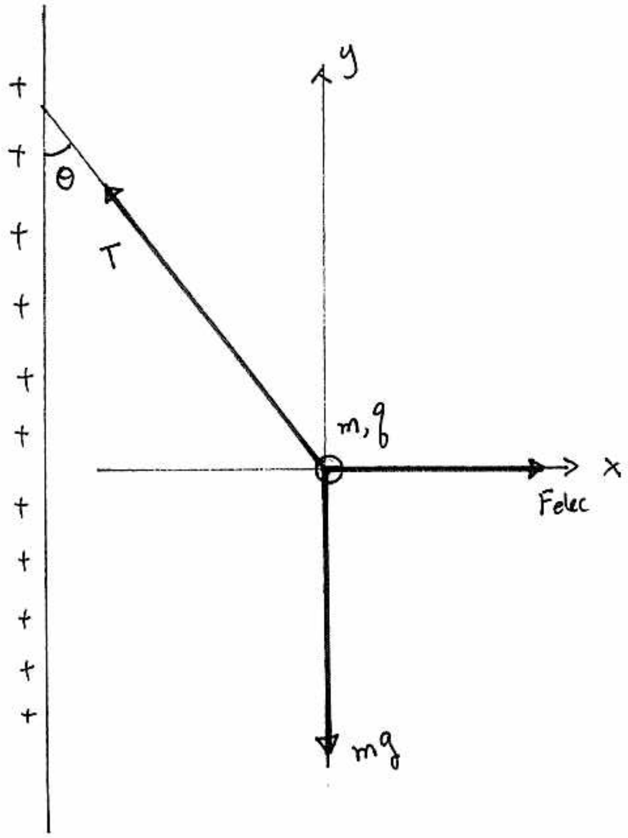
(b) for $r > R$ $\quad \begin{pmatrix} r = 0.05m \\ R = 0.04m \end{pmatrix}$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$
$$E \cdot 2\pi r L = \frac{\int_0^R dq}{\varepsilon_0} \longrightarrow$$

Gaussian Surface is outside the cylinder.
We need to integrate from 0 to the surface of the cylinder

$$E \cdot 2\pi r L = \frac{A\pi L R^4}{2\varepsilon_0}$$
$$\therefore\; E = \frac{AR^4}{4r\varepsilon_0} = \frac{2.5\times 10^{-6}\;\;(0.04)^4}{4\,(0.05)\;\varepsilon_0} = \underline{\underline{3.615819209\ \text{N/C}}}$$

#39



$m = 1.0\ mg = 1.0 \times 10^{-6} kg$

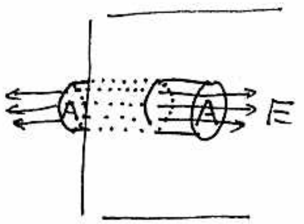
$q = 2 \times 10^{-8} C$

$\theta = 30°$

$F_{elec} = qE$

$\Sigma \vec{F} = \vec{T} + \vec{mg} + \vec{F_{elec}} = 0.$

First, $\vec{E}$ for a large sheet



$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2A = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore\ E = \frac{\sigma}{2\varepsilon_0} \quad — ①$$

Second, Analysis of Force.

<u>X comp</u>

$F_{elec} - T \sin 30° = 0$

$T \sin 30° = F_{elec} \quad — ②$

<u>y comp</u>

$T \cos 30° - mg = 0$

$T \cos 30° = mg \quad — ③$

$\frac{②}{③} \leftarrow ①$

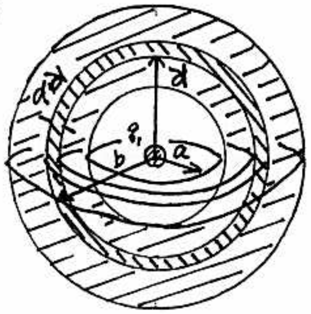
$$\frac{T \sin 30°}{T \cos 30°} = \frac{F_{elec}}{mg}$$

$$\tan 30° = \frac{qE}{mg} = \frac{q \frac{\sigma}{2\varepsilon_0}}{mg}$$

Solve for σ

$$\sigma = \frac{mg \tan 30°}{q\, 2\varepsilon_0} = \frac{(1 \times 10^{-6} kg)(9.81\ m/sec^2) \tan 30°}{(2 \times 10^{-8} c) \cdot 2 \cdot 8.85 \times 10^{-12}} = \underline{\underline{5.012468435 \times 10^{-9}\ C/m^2}}$$

#47



Sorry about the bad drawing

q_r in the shell from a up to r(<b)

Take a thin shell with radius, γ and a thickness $d\gamma$

$dq = \rho \cdot d(\text{vol})$ of a thin shell

$= \rho \cdot (4\pi \gamma^2 \cdot d\gamma)$ $\quad (\rho = \frac{A}{\gamma}$: Given)

$= \frac{A}{\gamma} (4\pi \gamma^2 \cdot d\gamma)$

$= 4\pi A \gamma \cdot d\gamma$

$$q_r = \int dq = \int_a^r 4\pi A \gamma \cdot d\gamma$$

$= 2\pi A (r^2 - a^2)$ — ① (charge in the shell up to r)

Gauss' Law

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_1 + q(\text{in the shell up to } r)}{\varepsilon_0} \text{ — ②}$$

② ⟵ ①

$$E = \frac{q_1 + 2\pi A (r^2 - a^2)}{4\pi \varepsilon_0 r^2}$$

the other condition given to this problem is that E is constant in the shell $(a \le r \le b)$ ⇒ $\frac{dE}{dr} = 0$

$$\frac{dE}{dr} = \frac{d\left(\frac{q_1 + 2\pi A(r^2 - a^2)}{4\pi \varepsilon_0 r^2}\right)}{dr} = \frac{d\left(\frac{q_1}{4\pi\varepsilon_0 r^2} + \frac{A}{2\varepsilon_0} - \frac{Aa^2}{2\varepsilon_0 r^2}\right)}{dr}$$

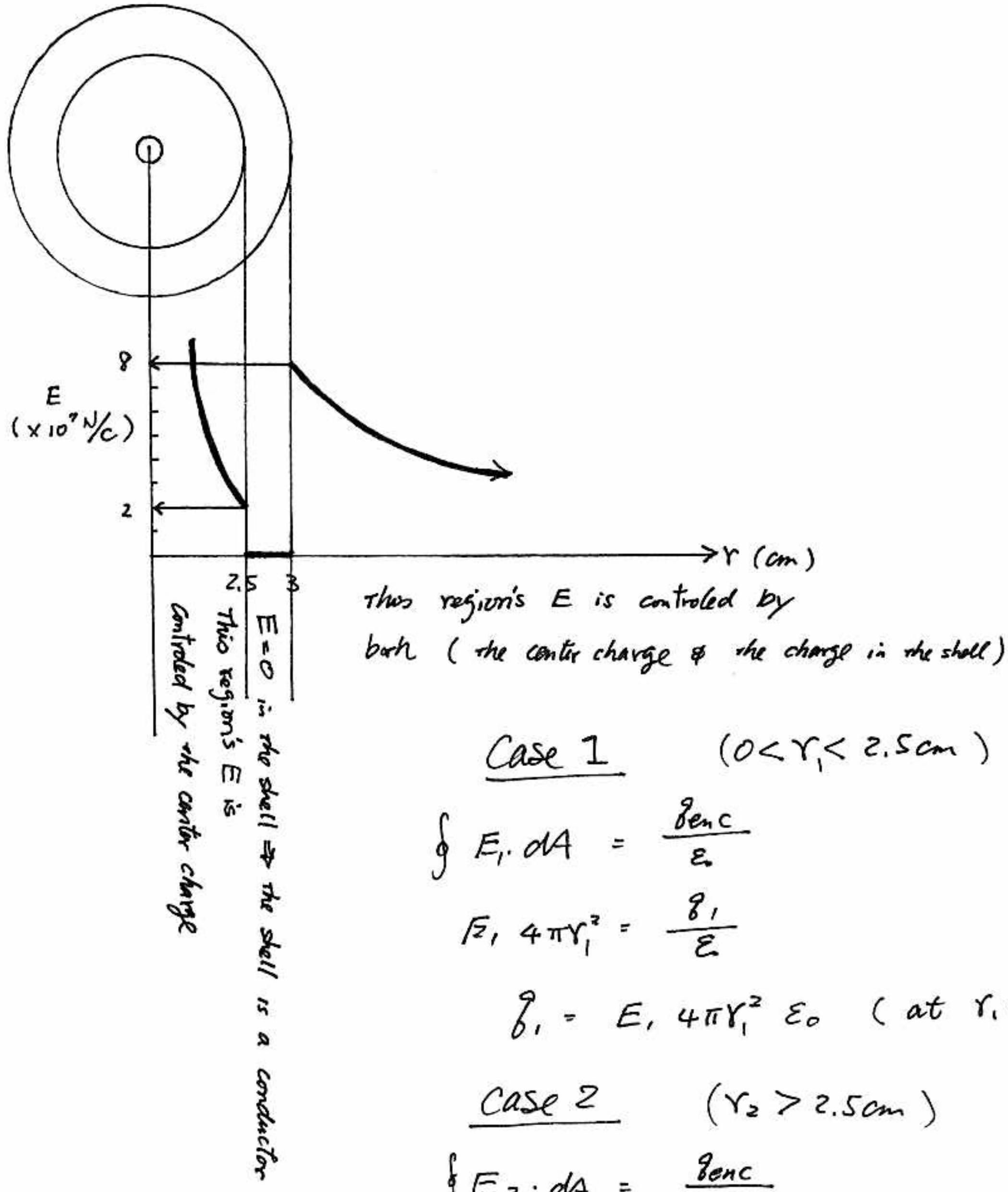
$$= \frac{-2q_1}{4\pi\varepsilon_0 r^3} - \frac{-2Aa^2}{2\varepsilon_0 r^3}$$

$$= \frac{-q_1 + 2\pi A a^2}{2\pi \varepsilon_0 r^3} = 0$$

$$\therefore \quad -q_1 + 2\pi A a^2 = 0$$

$$A = \frac{q_1}{2\pi a^2} = \frac{45\,fc}{2\pi\,(0.02m)^2} = \frac{45\times10^{-15}C}{2\pi\,(0.02m)^2} = \underline{\underline{1.79049311\times10^{-11}\ C/m^2}}$$

#48



<u>Case 1</u> $(0 < r_1 < 2.5\text{cm})$

$$\oint E_1 \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_1\, 4\pi r_1^2 = \frac{q_1}{\varepsilon}$$

$$q_1 = E_1\, 4\pi r_1^2\, \varepsilon_0 \quad (\text{at } r_1 = 2.5\text{cm},\ E_1 = 2\times10^{7}\,\text{N/C}) \quad ①$$

<u>Case 2</u> $(r_2 > 2.5\text{cm})$

$$\oint E_2 \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E_2 \cdot 4\pi r_2^2 = \frac{q_1 + q_{shell}}{\varepsilon_0} \quad \text{———} \quad ②$$

② ← ①

$$E_2 \cdot 4\pi r_2^2 = \frac{E_1\, 4\pi r_1^2 \varepsilon_0 + q_{shell}}{\varepsilon_0}$$

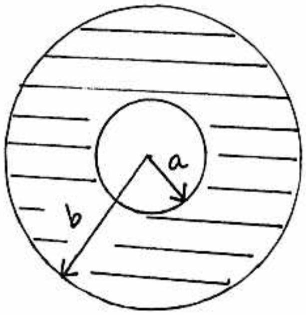
$$\therefore\ q_{shell} = E_2\, 4\pi r_2^2 \varepsilon_0 - E_1\, 4\pi r_1^2 \varepsilon_0 \qquad (\text{at } r_2 = 3.0\text{cm},\ E_2 = 8\times10^{7}\,\text{N/C})$$

$$= (E_2 r_2^2 - E_1 r_1^2)\, 4\pi \varepsilon_0$$

$$= \left[8\times10^{7}\,\text{N/C}\,(0.03\text{m})^2 - 2\times10^{7}\,\text{N/C}\,(0.025\text{m})^2\right] 4\pi \varepsilon_0$$

$$= \underline{\underline{6.617136606\times10^{-6}\,\text{C}}}$$

#50



$\rho = 1.84\ nC/m^3 \quad = \quad 1.84 \times 10^{-9}\ C/m^3$

$a = 10.0\ cm \quad = \quad 0.1\ m$

$b = 2a = 20.0\ cm \quad = 0.2\ m$

(a) $r = 0$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$ (because there is no - charge insde)

(b) $r = \frac{a}{2} = 0.05m$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$ (still there is no charge inside)

(c) $r = a$

$$E \cdot 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$\therefore \underline{\underline{E = 0}}$

(d) $r = 1.5a$

$$E \cdot 4\pi r^2 = \frac{\int_a^{1.5a} \rho \cdot 4\pi \partial^2 \cdot d\partial}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{4\pi\rho}{3\varepsilon_0}\left[(1.5a)^3 - (a)^3\right]$$

$$\therefore E = \frac{\rho}{3\varepsilon_0} \frac{(1.5a)^3 - (a)^3}{(1.5a)^2} = \underline{\underline{7.315337937\ N/C}}$$

(e) $r = 2a$ (use the same eqn. used in (d))

$$E = \frac{\rho}{3\varepsilon_0} \frac{(2a)^3 - (a)^3}{(2a)^2} = \underline{\underline{12.12806026\ N/C}}$$

(f) $r = 3b$

$$E \cdot 4\pi r^2 = \frac{\int_a^b \rho 4\pi \partial^2 \cdot d\partial}{\varepsilon_0} = \frac{4\pi\rho}{3\varepsilon_0}(b^3 - a^3)$$

$$\therefore E = \frac{\rho}{3\varepsilon_0 (3b)^2}(b^3 - a^3) = \underline{\underline{1.347562252\ N/C}}$$

#53

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \; 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$\therefore \; E = \frac{q_{enc}}{4\pi \varepsilon r^2} = K r^2 \quad \text{(Given condition)}$$

$$\therefore \; q_{enc} = 4\pi \varepsilon_0 K r^6 \; \text{———} \; ①$$

Also from the given condition,

"A charge distribution is spherically symmetric" — within a thin shell, the charge density is uniform.

$$\Rightarrow \; dq_{(\text{thin shell})} = \rho \cdot 4\pi \partial^2 \cdot d\partial$$

"... but not uniform radially" — so ρ is a variable charge density and is a fcn of radius

$$\Rightarrow \; \rho \propto \partial^n$$

(nth power since we do not know of ρ is proportional to ∂^1, ∂^2, ∂^3, or what)

$\therefore$ Let $\rho = \mathfrak{M} \partial^n$ where $\mathfrak{M}$ is some constant (could use "K" because "K" is already taken.)

$$\therefore \; dq = \mathfrak{M} \partial^n \cdot 4\pi \partial^2 \cdot d\partial = \mathfrak{M} \, 4\pi \partial^{n+2} \cdot d\partial$$

$$\therefore \; q_{enc} = \int_0^r \mathfrak{M} 4\pi \partial^{n+2} \cdot d\partial$$

$$= \frac{\mathfrak{M} \, 4\pi}{n+3} r^{n+3} \; \text{———} \; ②$$

$$① = ②$$

$$q_{enc} = 4\pi \varepsilon_0 K r^6 = \mathfrak{M} \frac{4\pi}{n+3} r^{n+3}$$

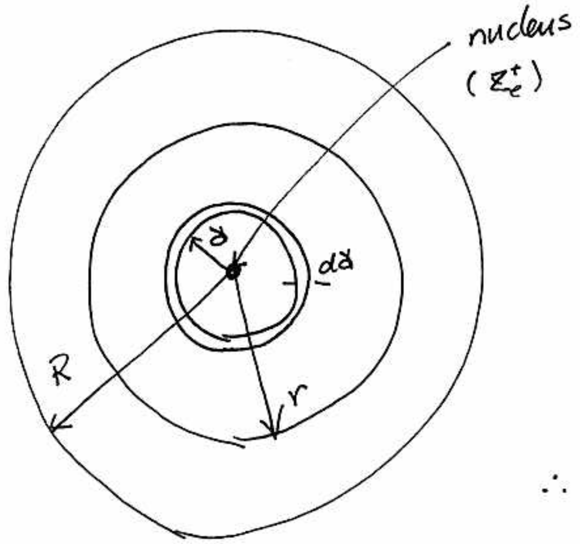
$$\therefore \; n = 3$$

$$\varepsilon_0 K = \frac{\mathfrak{M}}{3+3}$$

$$\mathfrak{M} = 6 \varepsilon_0 K$$

$$\therefore \; \underline{\rho = 6 \varepsilon_0 K r^3}$$

#83



$$\oint E\cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{\overset{\text{charge of the nucleus}}{Ze} + \overbrace{\int_0^r \rho\cdot 4\pi\partial^2\cdot d\partial}^{\text{charge of electron cloud up to 'r'}}}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Ze + \rho\frac{4\pi r^3}{3}}{\varepsilon_0}$$

$$\rho = \frac{\text{total charge}}{\text{Vol}} = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

$$\therefore\ E\cdot 4\pi r^2 = \frac{Ze + \left(\frac{-Ze}{\frac{4}{3}\pi R^3}\cdot\frac{4\pi r^3}{3}\right)}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Ze - Ze\frac{r^3}{R^3}}{\varepsilon_0}$$

$$\underline{\underline{E = \frac{Ze}{4\pi\varepsilon}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)}}$$

Extra

$$\rho = \frac{\rho_s\partial}{R}$$

(a) dq (of the shell) $= \rho\cdot 4\pi\partial^2\cdot d\partial$

$$= \frac{\rho_s\partial}{R}4\pi\partial^2\cdot d\partial$$

$$= \rho_s\frac{4\pi\partial^3}{R}\cdot d\partial$$

$$\therefore\ Q = \int dq = \int_0^R \rho_s\frac{4\pi\partial^3}{R}\cdot d\partial = \frac{4\pi\rho_s}{R}\cdot\frac{1}{4}\partial^4\Big|_0^R = \frac{\pi\rho_s R^4}{R}$$

$$= \underline{\underline{\pi\rho_s R^3}}$$

(b) Hence $\rho_s = \frac{Q}{\pi R^3}$

Gauss' Law

$$\oint E\cdot dA = \frac{q_{enc.}}{\varepsilon_0}$$

$$\varepsilon_0 E\cdot 4\pi r^2 = \int_0^r \rho\cdot 4\pi\partial^2\cdot d\partial = \int_0^r\frac{\rho_s\partial}{R}\cdot 4\pi\partial^2\cdot d\partial = \int_0^r\frac{\rho_s}{R}4\pi\partial^3\cdot d\partial$$

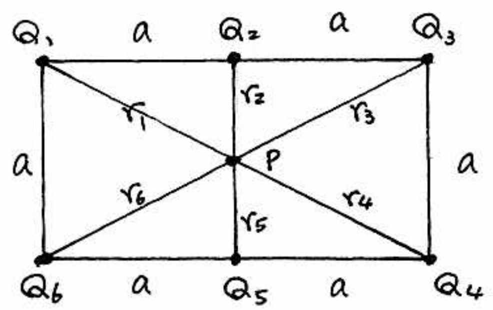
$$\varepsilon_0 E\, 4\pi r^2 = \frac{4\pi\rho_s}{R}\cdot\frac{1}{4}\partial^3\Big|_0^r = \frac{\pi\rho_s}{R}r^4$$

$$\therefore\ E = \frac{\cancel{\pi}\rho_s r^4}{4\cancel{\pi}\varepsilon_0 R r^2} = \frac{\frac{Q}{\pi R^3}r^2}{4\varepsilon_0 R}$$

$$= \underline{\frac{Q}{4\pi\varepsilon_0}\frac{r^2}{R^4}}$$

ch 24 #16, 21, 28, 29, 34, 35, 36, 37, 104, 114

#16



$Q_1 = Q_4 = +2q_1$

$Q_2 = Q_5 = +4q_2$

$Q_3 = -3q_1$

$Q_6 = -q_1$

$r_1 = r_3 = r_4 = r_6 = \sqrt{3}\,a$

$r_2 = r_5 = \frac{1}{2}a$

$$V = \Sigma V_i = \Sigma \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

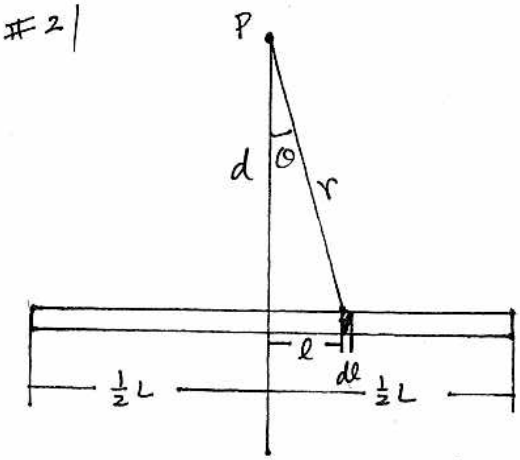
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1}+\frac{Q_2}{r_2}+\frac{Q_3}{r_3}+\frac{Q_4}{r_4}+\frac{Q_5}{r_5}+\frac{Q_6}{r_6}\right)$$

$$= \frac{1}{4\pi\varepsilon}\left[\left(\frac{Q_1+Q_3+Q_4+Q_6}{\sqrt{3}\,a}\right)+\left(\frac{Q_2+Q_5}{\frac{1}{2}a}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{+2q_1-3q_1+2q_1-q_1}{\sqrt{3}\,a}\right)+\left(\frac{4q_2+4q_2}{\frac{1}{2}a}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{16q_2}{a} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16\cdot 6\times10^{-12}C}{0.39m} = \underline{\underline{2.213367309 \text{ volts}}}$$

#21



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$dq = \lambda\, dl$

$r = (l^2+d^2)^{1/2}$

$$\therefore\ dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}}$$

(a)

$$V = \int dV = \int_{-\frac{1}{2}L}^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}} = 2\int_0^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(l^2+d^2)^{1/2}}$$

trig sub

Let $\tan\theta = \frac{l}{d}$

$l = d\tan\theta$

$dl = d\sec^2\theta\cdot d\theta$

$$\therefore\ V = 2\int_0^{\frac{1}{2}L}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, d\sec^2\theta}{(d^2\tan^2\theta+d^2)^{1/2}} = \frac{2\lambda}{4\pi\varepsilon_0}\int_0^{\frac{1}{2}L}\frac{d\sec^2\theta\cdot d\theta}{d\sec\theta}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\frac{1}{2}L}\sec\theta\cdot d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left|\sec\theta + \tan\theta\right| = \frac{\lambda}{2\pi\varepsilon_0} \ln\left|\frac{r}{d} + \frac{l}{d}\right| = \frac{\lambda}{3\pi\varepsilon} \ln\left|\frac{(l^2+d^2)^{1/2}+l}{d}\right|\Bigg|_0^{\frac{1}{2}L}$$

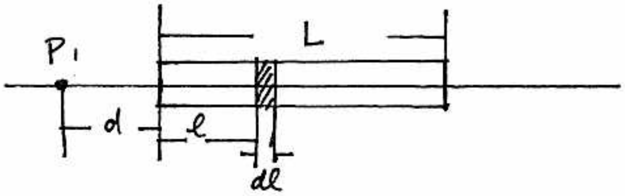
$$= \frac{3.68\times 10^{-12}\ C/m}{2\pi\ \varepsilon_0} \ln\left| \frac{((\frac{1}{2}L)^2+d^2)^{1/2}+\frac{1}{2}L}{d}\right|\Bigg|_0^{\frac{1}{2}L}$$

$L =$ cm
$d =$
$\lambda = 3.68\ pC/m$

$$= \underline{\underline{2.4269717\times 10^{-2}\ V}}$$

(b) $V = 0$ $\qquad |V_{by(+)}| = |V_{by(-)}|$

28



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\cdot dl}{(d+l)}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(d+l)}$$

$$= \frac{\lambda}{4\pi\varepsilon} \ln(d+l)\Big|_0^L$$

$$= \underline{\underline{\frac{\lambda}{4\pi\varepsilon_0} \ln\frac{(d+l)}{d}}}$$

plus in the values given

$$= \underline{7.389452298\times 10^{-3}\ \text{volts}}$$

$Q = 56.1\ fC = 56.1\times 10^{-15}\ C$
$L = 0.12m$
$d = 0.025m$

#29 using x instead of l

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dX}{(X+d)} \qquad \text{where } \lambda = cX$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{cX\, dx}{(X+d)}$$

$$= \frac{c}{4\pi\varepsilon_0}\frac{x}{(x+d)}dx$$

$$V = \int dV = \int_0^L \frac{c}{4\pi\varepsilon_0}\frac{x}{(X+d)}dx$$

Let $u = x + d \rightarrow x = u - d$

$du = dx$

$$V = \int \frac{c}{4\pi\varepsilon_0} \frac{u-d}{u} du$$

$$= \frac{c}{4\pi\varepsilon_0} \int (1 - \frac{d}{u}) du$$

$$= \frac{c}{4\pi\varepsilon_0} \quad u - d \ln u \Big|$$

$$= \frac{c}{4\pi\varepsilon} \left[(x+d) - d \ln(x+d)\right] \Big|_0^L$$

$$= \frac{c}{4\pi\varepsilon_0} \{[(L+d) - d \ln(L+d)] - [d - d \ln d]\}$$

$$= \frac{c}{4\pi\varepsilon_0} [L - d\{\ln(L+d) - \ln d\}]$$

$$= \frac{c}{4\pi\varepsilon_0} [L - d \ln(\frac{L+d}{d})]$$

w/ $L = 0.12\ m$, $C = 28.9\ pC/m^2$, $d = 0.03\ m$

$$= \frac{28.9 \times 10^{-12}}{4\pi\ \varepsilon_0} [0.12 - 0.03 \ln(\frac{0.12 + 0.03}{0.03})]$$

$$= \underline{\underline{1.863657 \times 10^{-2} \text{ volts}}}$$

#34

(a) $V = \frac{Q}{4\pi\varepsilon L} \ln(\frac{d+L}{d})$ (from #28 .. d is a positive number)

$$V = \frac{43.6 \times 10^{-15} c}{4\pi\varepsilon_0 (0.135 m)} \ln(\frac{d + 0.135}{d})$$

$$= \underline{\underline{2.904019886 \times 10^{-3} \ln(\frac{d + 0.135}{d})}}$$

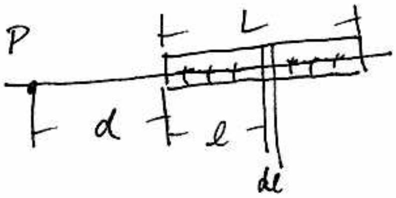
(b) $V = \frac{Q}{4\pi\varepsilon_0 L} \ln(\frac{x+L}{x})$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L} \frac{x}{x+L} \cdot (-\frac{L}{x^2})$$

$$= \underline{\underline{\frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{x(x+L)}}}$$

where x is the distance between the point & the beginning of the rod (at 0,0)

Just for a quick check



$$dE = \frac{1}{4\pi\varepsilon}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon}\frac{\lambda dx}{x^2}$$

$$E = \int dE = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(-\frac{1}{x}\right)\Big|_{d}^{d+L}$$

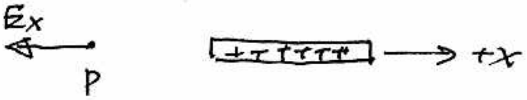
$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{d} - \frac{1}{d+L}\right)$$

$$= \frac{\frac{Q}{L}}{4\pi\varepsilon_0}\left(\frac{(d+L)-d}{d(d+L)}\right)$$

$$= \frac{Q}{4\pi\varepsilon L}\;\frac{L}{d(d+L)}$$

$$= \frac{Q}{4\pi\varepsilon_0}\;\frac{1}{d(d+L)}$$

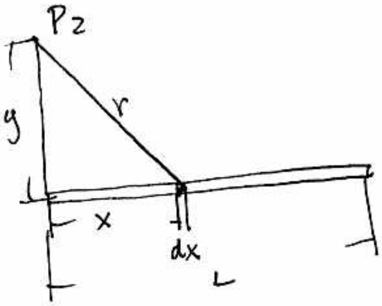
(c) the direction of E_x is negative



(d) $E = \frac{43.6\times10^{-15}}{4\pi\ \varepsilon_0}\cdot\frac{1}{0.062(0.062+0.135)} = \underline{\underline{3.2097812\times10^{-2}\ N/C}}$

(e) symmetry? there's no symmetry in y-direction. $dE_y = 0$ from any dq at P_1. $\therefore E_y = 0$.

#36 (since we are still using the same diagram)



(a) $$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$ $dq = \lambda dx$
$= cx\,dx$

$$= \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{(y^2+x^2)^{1/2}}$$

$$V = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{(y^2+x^2)^{1/2}}$$

Let $u = y^2+x^2$ $x=0 \rightarrow u=y^2$

$du = 2x\,dx$ $x=L \rightarrow u=y^2+L^2$

$$= \frac{C}{4\pi\varepsilon_0}\int\frac{\frac{1}{2}du}{u^{1/2}}$$

$$= \frac{c}{4\pi\varepsilon_0}\;u^{1/2}\Big|_{y^2}^{y^2+L^2}$$

$$= \underline{\underline{\frac{c}{4\pi\varepsilon_0}\left[(y^2+L^2)^{1/2} - y\right]}}$$

w/ the given values

$$= \frac{49.9\ \text{pC/m}}{4\pi\varepsilon_0}\left[\left((0.0356)^2 + (0.1)^2\right)^{1/2} - 0.0356\right] = \underline{\underline{3.1654176 \times 10^{-2}\ V}}$$

(b) $$E_y = -\frac{\partial V}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\left(\frac{1}{2}(y^2+L^2)^{-\frac{1}{2}} \cdot 2y - 1\right)$$

$$= \underline{\underline{\frac{c}{4\pi\varepsilon_0}\left(1 - \frac{y}{(y^2+L^2)^{1/2}}\right)}}$$

w/ the given values

$$= \frac{49.9\ \text{pC/m}}{4\pi\varepsilon_0}\left(1 - \frac{0.0356}{\left((0.0356)^2+(0.1)^2\right)^{1/2}}\right) = \underline{\underline{0.29820841\ \text{N/C}}}$$

(c) The V fcn we solved is a fcn of "y" although V could be a fcn of x. However, the information (to be able to describe V as a fcn of x) is not given. Hence we can not solve E_x by $-\frac{\partial V}{\partial x}$

\# 35 Since $E_x = -\frac{\partial V}{\partial x}$ & $E_y = -\frac{\partial V}{\partial y}$, E is a negative slope of V fcn. Fig 24-45. Both graphs show straight lines indicating E_x & E_y are constant (≡ slopes are constant)

$$E_x = -\frac{\partial V}{\partial x} = -\frac{-500\ V}{0.2\ m} = 2500\ \text{N/C}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{300\ V}{0.3\ m} = -1000\ \text{N/C}$$

$$\therefore\ \vec{E} = E_x\hat{i} + E_y\hat{j}$$

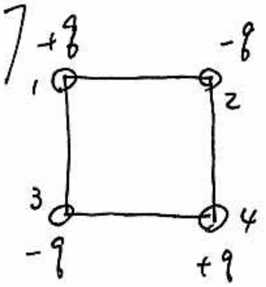
and $e^- = -1.6 \times 10^{-19}\ C$

$$\vec{F} = qE_x\hat{i} + qE_y\hat{j}$$

$$= (-1.6\times10^{-19}\ C)(2500\ \text{N/C})\hat{i} + (-1.6\times10^{-19}\ C)(-1000\ \text{N/C})\hat{j}$$

$$= \underline{\underline{-4\times10^{-16}\ N\,\hat{i} + 1.6\times10^{-16}\ N\,\hat{j}}} \quad (4.31\times10^{-16}\ N\ @\ -21.8^\circ)$$

#37



V created by other charges

$$W = \overset{\swarrow}{V} \cdot q$$

First, imagine there was no charge.

To bring the 1st charge,

$$W_1 = \cancel{V}^{0} q_1 \quad \Rightarrow \quad \underline{W_1 = 0}$$

2nd charge

$$W_2 = V_1 q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}(-q) = \underline{-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}}$$

3rd charge

$$W_3 = V_1 q_3 + V_2 q_3 = \underline{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{a}(-q) + \frac{1}{4\pi\varepsilon_0}\frac{-q}{\sqrt{2}a}(-q)}$$

4th charge

$$W_4 = V_1 q_4 + V_2 q_4 + V_3 q_4$$

$$= \underline{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{\sqrt{2}a}(+q) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{a}\right)(+q) + \frac{1}{4\pi\varepsilon_0}\frac{-q}{a}(+q)}$$

$$W_{Total} = \Sigma W_i$$

$$= 0 + \left[-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right] + \left[-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right] + \left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a} \times 2\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(-4 + \frac{2}{\sqrt{2}}\right)$$

$$= \underline{\underline{\frac{1}{4\pi\varepsilon}\frac{q^2}{a}\left(-4 + \sqrt{2}\right)}}$$

$$= \frac{1}{4\pi\varepsilon}\frac{(2.30\times10^{-12})^2}{(0.64m)}\left(-4+\sqrt{2}\right) = \underline{\underline{-1.921831098\times10^{-13}\,J}}$$

#104

(a) $$V = \frac{Ze}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)$$

$$E = -\frac{\partial V}{\partial r} = -\frac{Ze}{4\pi\varepsilon_0}\left(-\frac{1}{r^2} + \frac{2r}{2R^3}\right)$$

$$= \underline{\underline{\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)}}$$

(This is the ans. for #83, ch.23)

(b) where is $V = 0$?

$$V = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right) = 0$$

$$= \frac{Ze}{4\pi\varepsilon}\left(\frac{2R^3 - 3R^2r + r^3}{2rR^3}\right)$$

$$\therefore \quad 2R^3 - 3R^2r + r^3 = 0.$$

$$2R^3 - 2R^2r - R^2r + r^3 = 0$$

$$2R^2(R-r) - r(R^2-r^2) = 2R^2(R-r) - r(R+r)(R-r)$$

$$= (R-r)(2R^2 - r(R+r)) = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$r = R \qquad 2R^2 - rR - r^2 \text{ or } r^2 + Rr - 2R^2 = 0$$

$$r = \frac{-R \pm \sqrt{R^2 - 4(1)(-2R^2)}}{2}$$

$$r = \frac{-R \pm \sqrt{R^2 + 8R^2}}{2}$$

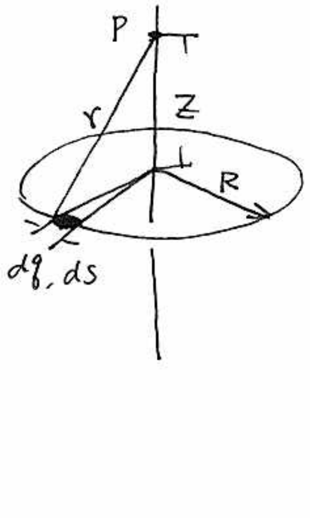
$$r = \frac{-R \pm 3R}{2} = \overset{\text{impossible}}{\cancel{-2R}} \text{ or } R$$

Either case $r = R$.

So, $V = 0$ was set at $r = R$ instead of $r = \infty$.
(You can set $V=0$ at anywhere you want. What's important is ΔV between two points. Not absolute V level.)

\# 114 $\quad V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} \qquad (24-32)$

(a)



$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$dq = \lambda\, ds$

$\lambda = \frac{Q}{2\pi R}$

$ds = R\, d\theta$

$r = (R^2 + Z^2)^{1/2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, d\theta}{(R^2+z^2)^{1/2}}$$

$$\therefore\ V = \int dV = \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0} \frac{\lambda R\, d\theta}{(R^2+z^2)^{1/2}}$$

$$= \frac{\lambda R}{4\pi\varepsilon_0} \frac{2\pi}{(R^2+z^2)^{1/2}}$$

$$= \frac{\frac{Q}{2\pi R}\cdot R}{4\pi\varepsilon_0} \frac{2\pi}{(R^2+z^2)^{1/2}}$$

$$= \underline{\underline{\frac{1}{4\pi\varepsilon_0} \frac{Q}{(R^2+z^2)^{1/2}}}}$$

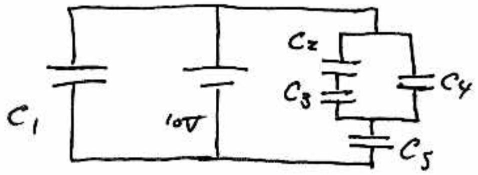
(b)

$$E = -\frac{\partial V}{\partial z} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{2} \frac{2z}{(R^2+z^2)^{3/2}}$$

$$= \underline{\underline{\frac{Q}{4\pi\varepsilon_0} \frac{z}{(R^2+z^2)^{3/2}}}}$$ ✓ (check w/ the eqn 22-16, P588)

ch 25 # 12, 23, 42, 43, 44

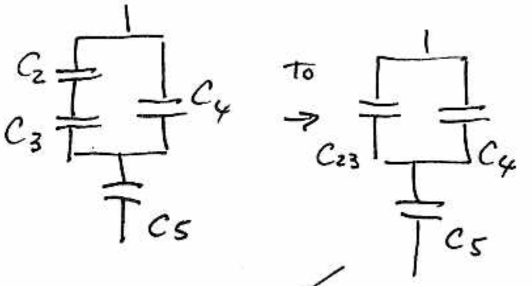
12



(a) $Q_1 = CV = 10\mu F \times 10V$

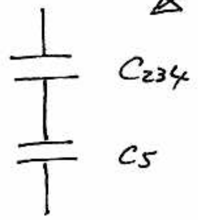
$= \underline{\underline{1 \times 10^{4} \, C}}$

(b) Before we can calculate C_2, we need to calculate C_{eq}.

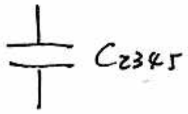


$$\underset{\text{(Series)}}{C_{23}} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_2 C_3}{C_2 + C_3} = \frac{10\mu F \cdot 10\mu F}{10\mu F + 10\mu F}$$

$$= 5\mu F$$



$$\underset{\text{(parallel)}}{C_{234}} = C_{23} + C_4 = 5\mu F + 10\mu F = 15\mu F$$



$$\underset{\text{(series)}}{C_{2345}} = \frac{1}{\frac{1}{C_{234}} + \frac{1}{C_5}} = \frac{1}{\frac{1}{15\mu F} + \frac{1}{10\mu F}} = \frac{15\mu F \cdot 10\mu F}{15\mu F + 10\mu F}$$

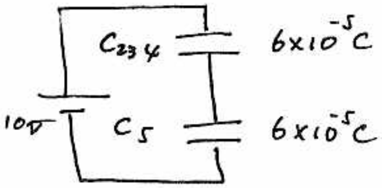
$$= \underline{\underline{6\mu F}}$$

Hence the Q in C_{eq} is

$Q = C_{eq} V = 6\mu F \times 10 V = 6 \times 10^{-5} C$

Now, going back to individual caps.

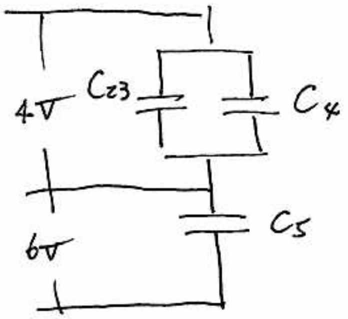
For series, $Q_{234} = Q_5 = 6 \times 10^{-5} C$



∴ V across C_{234} is

$$V_{(234)} = \frac{Q_{(234)}}{C_{(234)}} = \frac{6 \times 10^{-5} C}{15 \times 10^{6} F} = 4V$$

(The total Voltage drop across C_{234} & C_5 is 10V (= the source voltage) Hence V_5 should be $10V - 4V = 6V$.
check this w/ $V = \frac{Q}{C} = \frac{Q_5}{C_5} = \frac{6 \times 10^{-5} C}{10 \times 10^{-6} F} = 6V$ ✓)



$$Q_{23} = C_{23} V_{23} = 5\times10^{-6}\,F \cdot 4V = \underline{\underline{2\times10^{-5}C}}$$

$$(V_{23} = V_4 = 4V)$$

Since $Q_2 = Q_3$ (series) $= Q_{23}$

$$\underline{\underline{Q_2 = 2\times10^{-5}C}}$$

check

$$V_2 + V_3 = V_4 = 4V$$

$$\frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$= \frac{2\times10^{-5}C}{10\mu F} + \frac{2\times10^{-5}C}{10\mu F} = 2V + 2V = 4V \checkmark$$

Also,

$$Q_{23} + Q_4 = 6\times10^{-5}C$$

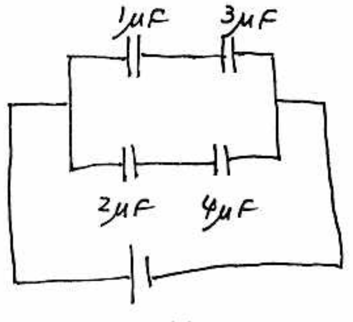
$$C_{23}V_{23} + C_4 V_4$$

$$= 2\times10^{-5}C + 10\mu F \cdot 4V$$

$$= 2\times10^{-5}C + 4\times10^{-5}C = 6\times10^{-5}C \checkmark$$

\# 23

(a)



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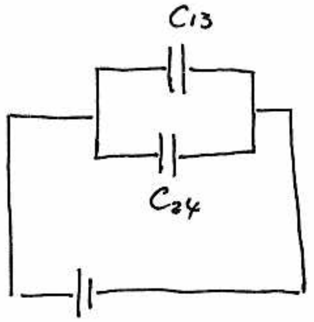


$$C_{13} = \frac{1}{\frac{1}{1}+\frac{1}{3}} = \frac{3}{4}\mu F$$

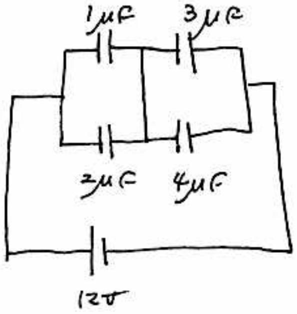
$$C_{24} = \frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{4}{3}\mu F$$

(d)

$$Q_{13} = C_{13}V_{13} = \frac{3}{4}\mu F \cdot 12V = \underline{\underline{9\times10^{-6}C \; (= Q_1 = Q_3)}}$$

$$Q_{24} = C_{24}V_{24} = \frac{4}{3}\mu F \cdot 12V = \underline{\underline{16\times10^{-6}C \; (= Q_2 = Q_4)}}$$

(e) ~ (h)



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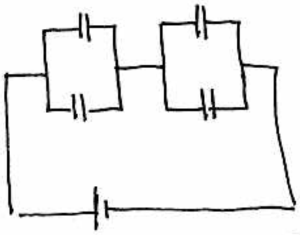

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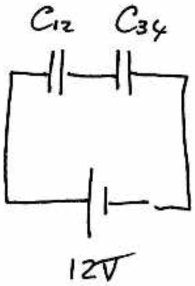
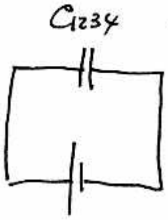


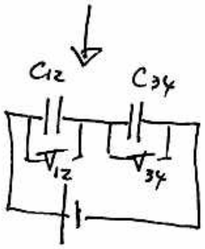
$$C_{12} = C_1 + C_2 = 1\mu F + 2\mu F = 3\mu F$$

$$C_{34} = C_3 + C_4 = 3\mu F + 4\mu F = 7\mu F$$



$$C_{1234} = \frac{1}{\frac{1}{3\mu F} + \frac{1}{7\mu F}} = \frac{21}{10}\ \mu F = \underline{2.1\mu F}$$

$$Q = CV = 2.1\mu F \cdot 12V = 25.2\mu C \quad (= Q_{12} = Q_{34})$$



$$V_{12} = \frac{Q_{12}}{C_{12}} = \frac{25.2\mu C}{3\mu F} = 8.4V$$

$$V_{34} = \frac{Q_{34}}{C_{34}} = \frac{25.2\mu C}{7\mu F} = 3.6V$$

$8.4V + 3.6V = 12V$

(equal to the source voltage)

$$Q_1 = C_1 V_1 = C_1 V_{12} = 1\mu F \cdot 8.4V = \underline{\underline{8.4\mu C}}$$

$$Q_2 = C_2 V_2 = C_2 V_{12} = 2\mu F \cdot 8.4V = \underline{\underline{16.8\mu C}}$$

$8.4\mu C + 16.8\mu C = 25.2\mu C$ ✓

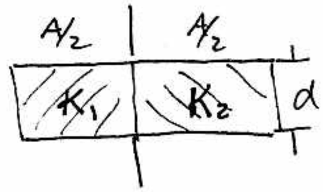
$$Q_3 = C_3 V_3 = C_3 V_{34} = 3\mu F \cdot 3.6V = \underline{\underline{10.8\mu C}}$$

$$Q_4 = C_4 V_4 = C_4 V_{34} = 4\mu F \cdot 3.6V = \underline{\underline{14.4\mu C}}$$

$10.8\mu C + 14.4\mu C = 25.2\mu C$ ✓

#42

Capacitance of a parallel plate



step 1 (E)

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E A = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0}$$

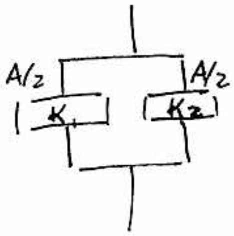
step 2 (V)

$$V = \int E \cdot dr = \int_0^d \frac{\sigma}{\varepsilon_0} dr = \frac{\sigma}{\varepsilon_0} d$$

step 3 (Q = CV ∴ $C = \frac{Q}{V}$)

$$C_0 = \frac{Q}{\frac{\sigma}{\varepsilon_0} d} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{A \varepsilon_0}{d}$$

Applying this to the question:



$$C_1 = K_1 \frac{(\frac{1}{2}A)\varepsilon_0}{d} = K_1 \frac{A\varepsilon_0}{2d}$$

$$C_2 = K_2 \frac{(\frac{1}{2}A)\varepsilon_0}{d} = K_2 \frac{A\varepsilon_0}{2d}$$

You can think as two parallel plate caps put them in parallel connection.

$$\therefore C_{eq} = C_1 + C_2 = K_1 \frac{A\varepsilon_0}{2d} + K_2 \frac{A\varepsilon_0}{2d}$$

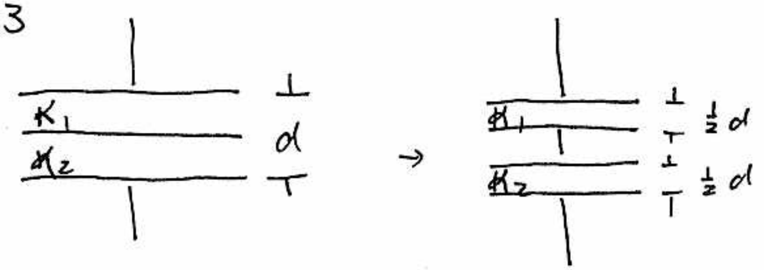
$$= \frac{A\varepsilon_0}{2d}(K_1 + K_2)$$

$$= \frac{5.56\,cm^2 \cdot \varepsilon_0}{2 \cdot 5.56\,mm}(7 + 12)$$ (Convert them into SI !!)

$$= \frac{5.56\,cm^2 \cdot \frac{1m^2}{(10^2)^2 cm^2} \cdot \varepsilon_0}{2 \cdot 5.56mm \cdot \frac{1m}{10^3 mm}}(7+1)$$

$$= \underline{\underline{8.4075 \times 10^{-12} F}}$$

#43



Two parallel plate caps connected in Series

Step 1 $E = \frac{\sigma}{\varepsilon_0}$ (see #42)

Step 2 $V = \int_0^{\frac{1}{2}d} E \cdot dr = \frac{\sigma}{\varepsilon_0}(\frac{1}{2}d)$

Step 3 $C = \frac{Q}{V} = \frac{Q}{\frac{\sigma}{\varepsilon_0}\frac{1}{2}d} = \frac{\sigma A}{\frac{\sigma d}{2\varepsilon_0}} = \frac{2A\varepsilon_0}{d}$

$\therefore C_1 = \frac{2K_1 A\varepsilon_0}{d}$ & $C_2 = \frac{2K_2 A\varepsilon_0}{d}$

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{2K_1 A\varepsilon_0}{d} \cdot \frac{2K_2 A\varepsilon_0}{d}}{\frac{2K_1 A\varepsilon_0}{d} + \frac{2K_2 A\varepsilon_0}{d}}$$

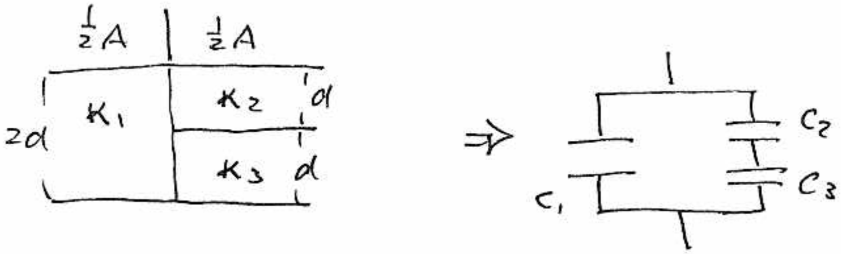
$$= \frac{2K_1 A \varepsilon_0 \, 2K_2 A\varepsilon_0}{d(2K_1 A\varepsilon_0 + 2K_2 A\varepsilon_0)}$$

$$= \frac{4A^2\varepsilon_0^2 K_1 K_2}{2A\varepsilon_0 d(K_1+K_2)}$$

$$= \frac{2A\varepsilon_0}{d} \frac{K_1 K_2}{K_1+K_2} = \frac{2 \cdot 7.89\,cm^2 \cdot \varepsilon_0}{4.62mm} \cdot \frac{11 \cdot 12}{11+12}$$

$$= \frac{2 \cdot 7.89\times10^{-4}m^2 \cdot \varepsilon_0}{4.62\times10^{-3}m} \cdot \frac{11 \cdot 12}{11+12} = \underline{\underline{1.734819876\times10^{-11}F}}$$

#44 this is a combination of #42 & 43



$$C_1 = \frac{\kappa_1 \cdot \frac{1}{2} A \varepsilon_0}{2d} , \quad C_2 = \frac{\kappa_2 \cdot \frac{1}{2} A \varepsilon_0}{d} , \quad C_3 = \frac{\kappa_3 \cdot \frac{1}{2} A \varepsilon_0}{d}$$

$$= \frac{\kappa_1 A \varepsilon_0}{4d} , \quad = \frac{\kappa_2 A \varepsilon_0}{2d} , \quad = \frac{\kappa_3 A \varepsilon_0}{2d}$$

$$C_{23} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_2 C_3}{C_2 + C_3} = \frac{A \kappa_2 \kappa_3 \varepsilon_0}{2d(\kappa_2 + \kappa_3)}$$

(Series)

$$C_{123} = C_1 + C_{23}$$

(parallel)

$$= \frac{\kappa_1 A \varepsilon_0}{4d} + \frac{A \kappa_2 \kappa_3 \varepsilon_0}{2d(\kappa_2 + \kappa_3)}$$

$$= \frac{A \varepsilon_0 (\kappa_1 (\kappa_2 + \kappa_3) + 2 \kappa_2 \kappa_3)}{4d(\kappa_2 + \kappa_3)}$$

$$= \frac{A \varepsilon_0 (\kappa_1 \kappa_2 + 2 \kappa_2 \kappa_3 + \kappa_3 \kappa_1)}{4d(\kappa_2 + \kappa_3)} = \frac{10.5\,\mathrm{cm}^2 \cdot \varepsilon_0 (21 \cdot 42 + 2 \cdot 42 \cdot 58 + 58 \cdot 21)}{4\left(\frac{7.12\,\mathrm{mm}}{2}\right)(42 + 58)}$$

$$= \frac{10.5 \times 10^{-4}\,\mathrm{m}^2 \cdot \varepsilon_0 (21 \cdot 42 + 2 \cdot 42 \cdot 58 + 58 \cdot 21)}{2(7.12 \times 10^{-3})(42 + 58)} = \underline{\underline{4.54967064 6 \times 10^{-11}\,\mathrm{F}}}$$

ch. 26 #1, 2, 21, 43, 49

#1

(a) $i = 5.0\ A$, $t = 4\ min$

$$i = \frac{dq}{dt} \Rightarrow \int dq = \int i\, dt = i \cdot t$$

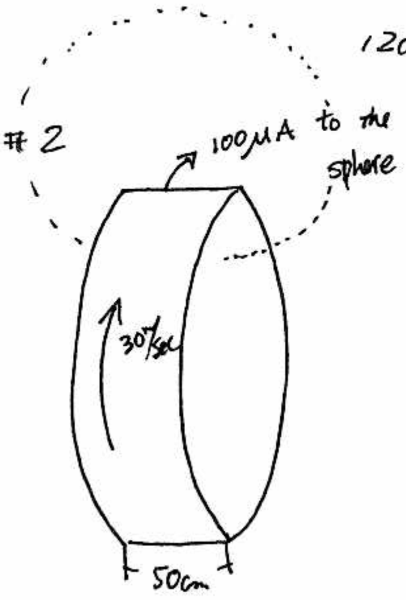
$$= (5\ A)(4\ min) = 5A \cdot 240\ sec$$

$$= \underline{\underline{1200\ C}}$$

(b) $e = 1.6 \times 10^{-19} C$

$$1200\ C \times \frac{1e}{1.6 \times 10^{-19} C} = \underline{\underline{7.5 \times 10^{21}\ \text{electrons}}}$$

#2



$$i = 100\ \mu A = 100 \times 10^{-6}\ C/sec$$

$$dq = \sigma \cdot dA$$

$$\frac{dq}{dt} = \frac{\sigma \cdot dA}{dt}$$

$$= \sigma \frac{d(W \cdot L)}{dt}$$

W: Width
L: length

$$= \sigma W \left(\frac{dL}{dt}\right)$$ (width is const. = 50cm)

$$= \sigma W \cdot Vel$$

$$\therefore\ i = \frac{dq}{dt} = \sigma W\ Vel$$

$$\therefore\ \sigma = \frac{i}{W \cdot Vel} = \frac{100 \times 10^{-6}\ C/sec}{(0.5m)(30m/sec)} = 6.\dot{6}\ C/m^2$$ (units match!)

21

$$R = \rho \frac{L}{A} = 6\ \Omega$$

$$\rho = 6\Omega \cdot \frac{A}{L}$$

New specifications Length → 3L
Area → $\frac{1}{3}A$

$\left(Vol = l_1 A_1 = l_2 A_2 \text{ if } l_2 = 3 l_1,\ A_2 = \frac{1}{3} A_1 \right)$

$$\therefore\ R_{New} = \rho \cdot \frac{3L}{\frac{1}{3}A} = \left(6\Omega \frac{A}{L}\right) \frac{3L}{\frac{1}{3}A}$$

$$= \underline{\underline{54\ \Omega}}$$

#43 $P = 100\,W$ cost = 6¢/Kwh

$V = 120\,V$

(a) Total energy consumption (in Kwh) in one month

$$100\,W \times 30\text{ days} \cdot \frac{24\text{ hrs}}{\text{day}} = 7.2 \times 10^4\,W\,hr$$
$$= 7.2 \times 10^1\,KW\,hr$$

Since the cost is 6¢/Kwh

$$7.2 \times 10^1\,Kwh \times 6¢/Kwh = 432¢ = \underline{\underline{\$\,4.32}}$$

(b)
$$V = I \cdot R \Rightarrow I = \frac{V}{R}$$
$$P = I \cdot V = \frac{V}{R} \cdot V = \frac{V^2}{R}$$
$$\therefore R = \frac{V^2}{P} = \frac{(120\,V)^2}{100\,W} = \underline{\underline{144\,\Omega}}$$

(c)
$$V = I \cdot R$$
$$I = \frac{V}{R} = \frac{120\,V}{144\,\Omega} = \underline{\underline{0.8\dot{3}\text{ amp}}}$$

#79 Eqn 26-17 : $\rho - \rho_0 = \rho_0 \alpha (T - T_0)$

$\rho_0 = 1.69 \times 10^{-8}\,\Omega m$

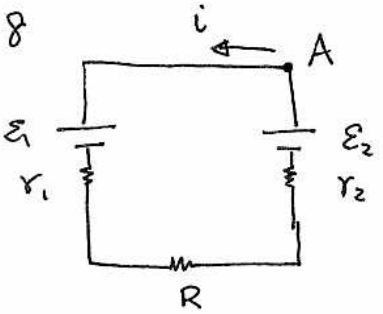
$T_0 = 20°C = 293\,K$

$\alpha = 4.3 \times 10^{-3}/K$

$$\rho = 2\rho_0$$
$$\therefore 2\rho_0 - \rho_0 = \rho_0 \alpha (T - T_0)$$
$$1 = \alpha (T - T_0)$$
$$T - T_0 = \frac{1}{\alpha}$$
$$T = \frac{1}{\alpha} + T_0 = \frac{1}{4 \times 10^{-3}/K} + 293\,K = 525.96\,K$$
$$= \underline{\underline{252.56°C}}$$

ch 27 #8, 18, 27, 41, 42, 52, 55, 67, 68, 89

#8



$\mathcal{E}_1 = 2V$

$\mathcal{E}_2 = 3V$

$r_1 = r_2 = 3\Omega$

$i = 1\times10^{-3}A$

Because $\mathcal{E}_2 > \mathcal{E}_1$, the direction of i is counter-clockwise.

(a) Loop (starting at A)

$$-\mathcal{E}_1 - ir_1 - iR - ir_2 + \mathcal{E}_2 = 0$$

$$-\mathcal{E}_1 + \mathcal{E}_2 = i(r_1 + R + r_2)$$

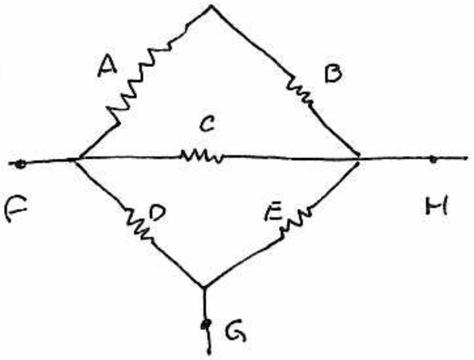
$$-2 + 3 = i(3 + R + 3)$$

$$1 = i(6 + R)$$

$$\therefore R = \frac{1}{i} - 6 = \frac{1}{1\times10^{-3}} - 6 = \underline{\underline{994\ \Omega}}$$

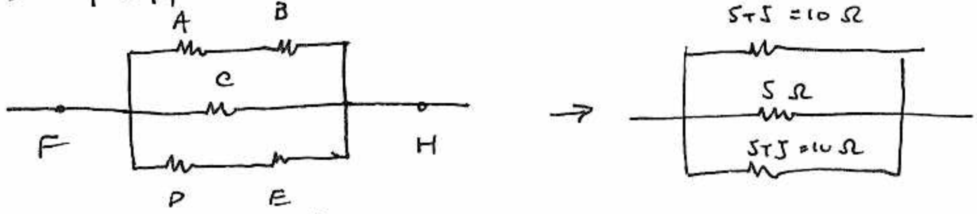
(b) $P = IV = I^2R = (1\times10^{-3})^2 \cdot 994\ \Omega = \underline{\underline{9.94\times10^{-4}\ W}}$

#18



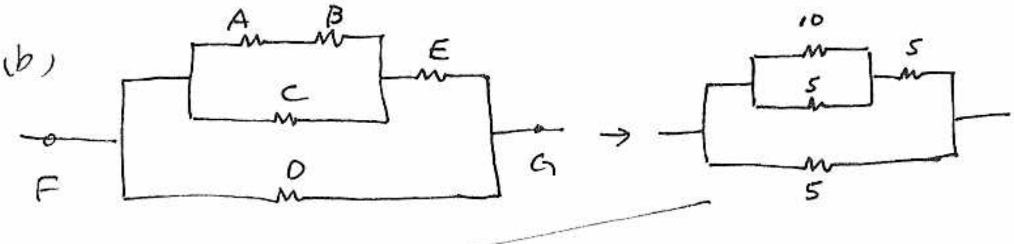
all R's are 5 Ω

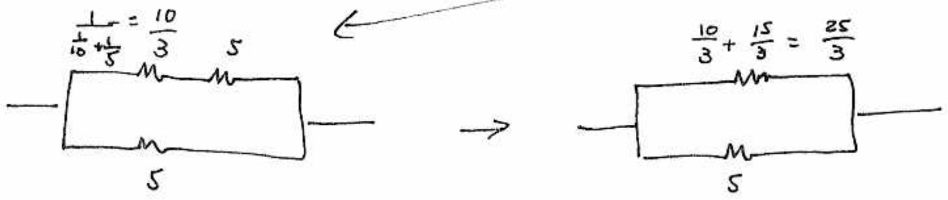
(a) F-H



→ $R_{eq} = \frac{1}{\frac{1}{10}+\frac{1}{5}+\frac{1}{10}} = \frac{10}{4} = \underline{\underline{2.5\ \Omega}}$

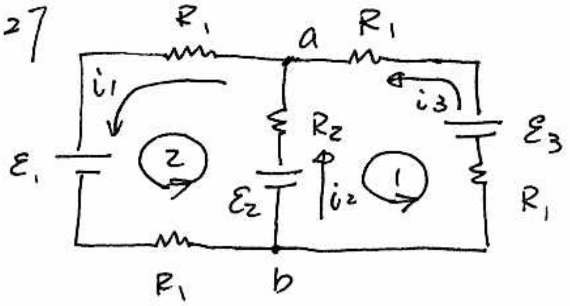
(b)





→ $R_{eq} = \frac{1}{\frac{3}{25}+\frac{1}{5}} = \underline{\underline{\frac{25}{8}\ \Omega}}$

27



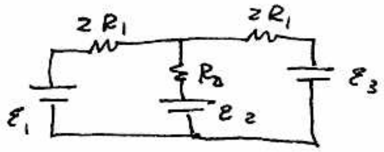
$\mathcal{E}_1 = 2V$

$\mathcal{E}_2 = \mathcal{E}_3 = 4V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

You can solve this by reducing first to



But I will do the harder way to show it still works.

Junction Rule (at a)

$i_2 + i_3 = i_1$ ——— ①

Loop 1 (at a)

$i_2R_2 - \mathcal{E}_2 - i_3R_1 + \mathcal{E}_3 - i_3R_1 = 0$

$2i_2 - 4 - i_3 + 4 - i_3 = 0$

$\therefore 2i_2 - 2i_3 = 0$

$\therefore i_2 = i_3$ ——— ②

Loop 2 (at a)

$-i_1R_1 - \mathcal{E}_1 - i_1R_1 + \mathcal{E}_2 - i_2R_2 = 0$

$i_1 - 2 - i_1 + 4 - 2i_2 = 0$

$-2i_1 + 2 - 2i_2 = 0$

$\therefore -i_1 + 1 - i_2 = 0$

$i_2 = 1 - i_1$ ——— ③

① ← ② ← ③

$(1 - i_1) + (1 - i_1) = i_1$

$2 - 2i_1 = i_1$

$3i_1 = 2$

(a)&(b) $\therefore i_1 = \frac{2}{3}$ amp — ①′ ↓

(c)&(d) ③ ← ①′

$i_2 = 1 - \frac{2}{3}$ ap

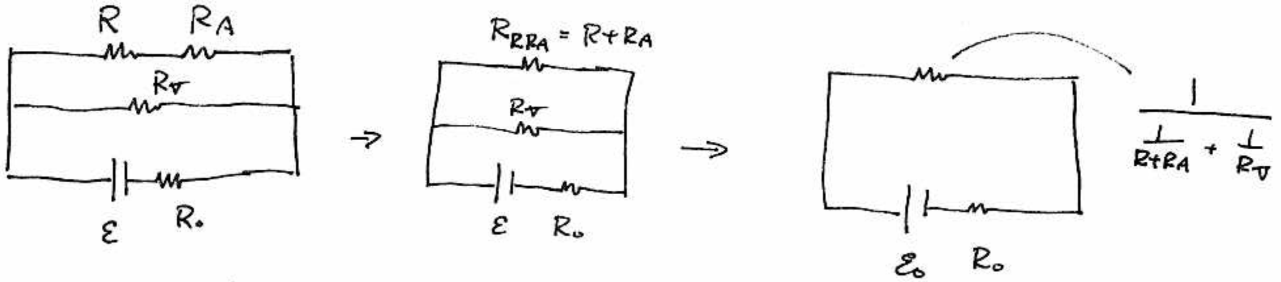
$= \frac{1}{3}$ ap — ③′ ↓

(e)&(f) ② ← ③′

$i_3 = \frac{1}{3}$ ap ↑

(g) $V_a - V_b = i_2R_2 - \mathcal{E}_2 = \frac{1}{3} \cdot 2 - 4 = -\frac{10}{3}V$

#41



(a) $R_T = \frac{R_V(R+R_A)}{R+R_A+R_V} + R_0 = \frac{300(85+3)}{300+85+3} + 100 = 168.0412371\,\Omega$

$$i_T = \frac{V_T}{R_T} = \frac{12V}{168.041..} = \underline{\underline{7.141104295\times10^{-2}\text{ amp}}}$$

$$V_{\left(\frac{1}{\frac{1}{R+R_A}+\frac{1}{R_V}}\right)} = i_T\, R_{\frac{1}{\frac{1}{R+R_A}+\frac{1}{R_V}}}$$

$$= 7.14 .. \frac{R_V(R+R_A)}{R+R_A+R_V} = \underline{4.8558895706\,V}$$

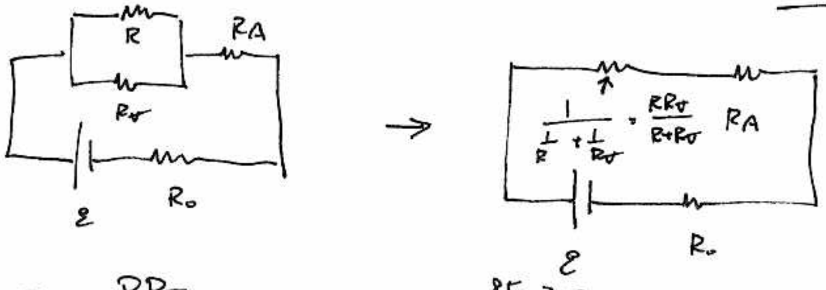
$$\therefore\ i_{(\text{trough } Ⓐ)} = \frac{V}{R} = \frac{4.855..}{85+3} = \underline{\underline{5.521472393\times10^{-2}\text{ ap}}}$$

(b) $V = V_{\left(\frac{1}{\frac{1}{R+R_A}+\frac{1}{R_V}}\right)} = \underline{\underline{4.8558895706\,V}}$

(c) $R' = \frac{V'}{i} = \frac{4.85..}{5.52..} = \underline{\underline{88\,\Omega}}$

(d) If $R_A\downarrow$, $R_T\downarrow$, $i_T\uparrow$ $V\downarrow$, $\therefore\ \underline{\underline{R' = \frac{V}{i}\downarrow}}$

#42



(a) $R_T = \frac{RR_V}{R+R_V} + R_A + R_0 = \frac{85\cdot300}{85+300} + 3 + 100 = 1.69\times10^{2}\,\Omega$

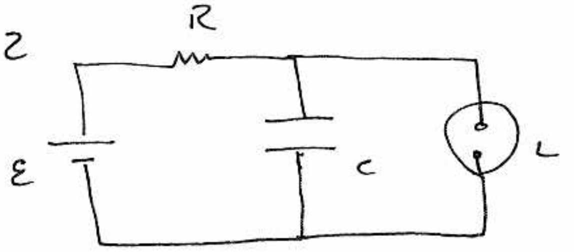
$$i = \frac{V}{R_T} = \frac{12}{1.69\times10^{2}} = \underline{\underline{7.09\times10^{-2}\text{ amp}}}$$

(b) $V = i R_{eq} = 209\times10^{-2}\text{ ap}\cdot\frac{RR_V}{R+R_V} = 4.696492978\,V$

(c) $R' = \frac{V}{i} = \frac{4.696\cdots}{7.09\times10^{-2}} = 66.23376623\,\Omega$ (way off from real R because R_V is too small)

(d) $R_V\uparrow$, $R_T\uparrow$ $i\downarrow$, $\underline{\underline{R' = \left(\frac{V}{i}\right)\uparrow}}$

52



$C = 0.15\,\mu F$

$\varepsilon = 95V$

$V_L = 72V$

2 Flashes / sec $\Rightarrow t = 0.5\,sec$

$V_C = \varepsilon(1 - e^{-\frac{t}{RC}})$ [You should be able to derive this]

$\frac{V_C}{\varepsilon} = (1 - e^{-\frac{t}{RC}})$

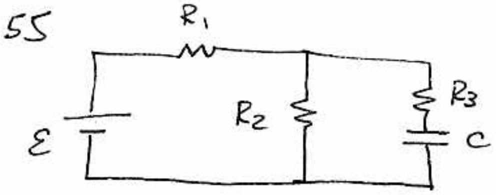
$e^{-\frac{t}{RC}} = 1 - \frac{V_C}{\varepsilon}$

$-\frac{t}{RC} = \ln(1 - \frac{V_C}{\varepsilon})$

$R = \frac{-t}{C\ln(1-\frac{V_C}{\varepsilon})}$ (at $t=0$, $V_C = 0$ & $t = 0.5\,sec$, $V_C = 72V$)

$= \frac{-0.5}{0.15\times10^{-6}(1-\frac{72}{95})} = \underline{\underline{2350094.506\,\Omega \quad (2.35\times10^{6}\Omega)}}$

55

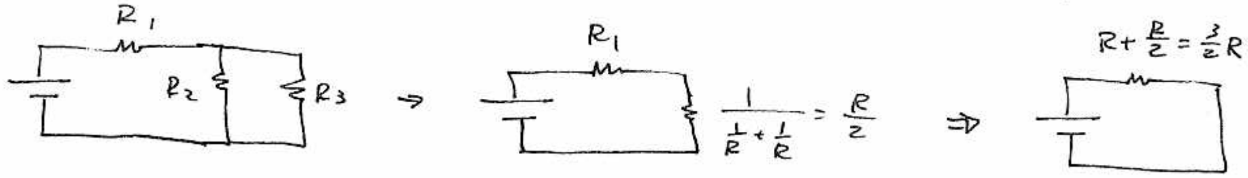


$\varepsilon = 1200V$

$C = 6.5\times10^{-6}F$

$R_1 = R_2 = R_3 = 0.73\times10^{6}\Omega$

(a), (b), & (c) when $t=0$, C is considered "short circuited":



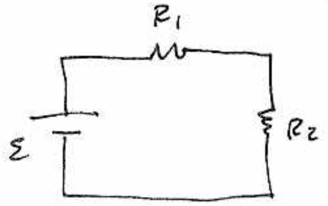
$\therefore i = \frac{V}{R_{eq}} = \frac{1200}{\frac{3}{2}R} = \underline{\underline{1.095890411\times10^{-3}\,amp}} \quad (= i_1)$

Because $R_2 = R_3$, the current will split half & half

$i_2 = i_3 = \underline{\underline{5.479452055\times10^{-4}\,amp}} \quad (= i_2 = i_3)$

(d), (e), & (f) when $t=\infty$, $\underline{\underline{i_3 = 0}}$ because C is fully charged.

So the circuit is considered as:

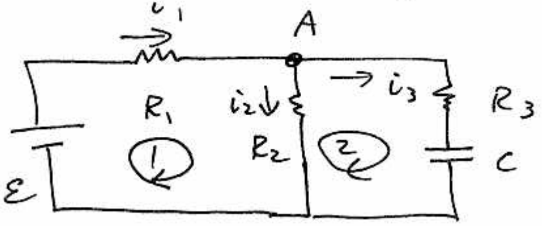


$R_{eq} = R_1 + R_2$

$i = \frac{V}{R_{eq}} = \underline{\underline{8.219178082\times10^{-4}\,amp}} \quad (i_1 = i_3)$

(g), (h), & (i)

You can calculate $V_2(t=0)$ by $i(t=0)R_2$ $(= 400\,V)$ and $V_2(t=\infty)$ by $i(t=\infty)R_2$ $(= 600\,V)$. But I want to show anorher way so that we can calalate i_2 as a fcn of time (which you should be able to do)



Junction Rule (at A)

$i_1 = i_2 + i_3$ ——— ①

$i_3 = \frac{dq}{dt}$ (the current that goes a capacitor is $\frac{dq}{dt}$

Loop 1 (at A)

$-i_2R_2 + \mathcal{E} - i_1R_1 = 0$ ——— ②

Loop 2 (at A)

$-i_3R - \frac{q}{C} + i_2R = 0$ ——— ③

② ← ①

$$-i_2R + \mathcal{E} - (i_2 + i_3)R = 0$$

$$-i_2R + \mathcal{E} - i_2R - i_3R = 0$$

$$-2i_2R - i_3R + \mathcal{E} = 0$$

$$2i_2R = \mathcal{E} - i_3R$$

$\therefore\ i_2 = \frac{\mathcal{E} - i_3R}{2R}$ ——— ②'

③ ← ②'

$$-i_3R - \frac{q}{C} + \left(\frac{\mathcal{E} - i_3R}{2R}\right)R = 0$$

$$-i_3R - \frac{q}{C} + \frac{\mathcal{E}}{2} - \frac{i_3R}{2} = 0$$

$-\frac{3}{2}i_3R - \frac{q}{C} + \frac{\mathcal{E}}{2} = 0$ [We want solve for i_3 since $i_3 = \frac{dq}{dt}$]

$$\therefore\ i_3 = \frac{2}{3R}\left(-\frac{q}{C} + \frac{\mathcal{E}}{2}\right) = \frac{2}{3R}\left(\frac{-2q + \mathcal{E}C}{2C}\right)$$

$$= \frac{-1}{3RC}(2q - \mathcal{E}C)$$

$$= \frac{-2}{3RC}\left(q - \frac{\mathcal{E}C}{2}\right) = \frac{dq}{dt}$$

$$\int \frac{-2}{3RC}\, dt = \int \frac{1}{q - \frac{\varepsilon C}{2}}\, dq$$

$$e^{\frac{-2t}{3RC}} = e^{\ln\left(q - \frac{\varepsilon C}{2}\right) + \gamma}$$

$$e^{-\frac{2t}{3RC}} = \left(q - \frac{\varepsilon C}{2}\right)\gamma'$$

when $t = 0,\ q = 0$

$$\therefore\ 1 = -\frac{\varepsilon C}{2}\gamma'$$

$$\therefore\ \gamma' = -\frac{2}{\varepsilon C}$$

So the fcn is

$$e^{\frac{-2t}{3RC}} = \left(q - \frac{\varepsilon C}{2}\right)\left(-\frac{2}{\varepsilon C}\right)$$

$$\therefore\ \underline{\underline{q = \frac{\varepsilon C}{2}\left(1 - e^{-\frac{2t}{3RC}}\right)}}$$

$$\text{So, } i_3 = \frac{dq}{dt} = \frac{\varepsilon C}{2}\,\frac{2}{3RC}\, e^{\frac{-2t}{3RC}} = \frac{\varepsilon}{3R} e^{\frac{-2t}{3RC}} \text{ ———— } ③'$$

$$②' \longleftarrow ③'$$

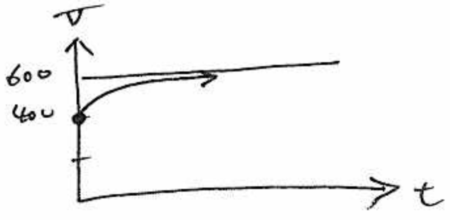
$$i_2 = \frac{\varepsilon - \left(\frac{\varepsilon}{3R} e^{\frac{-2t}{3RC}}\right)R}{2R} = \underline{\underline{\frac{\varepsilon}{2R}\left(1 - \frac{1}{3} e^{\frac{-2t}{3RC}}\right)}}$$

$$\therefore\ V_{(R_2)} = i_2 R_2$$

$$= \underline{\underline{\frac{\varepsilon}{2}\left(1 - \frac{1}{3} e^{-\frac{2t}{3RC}}\right)}}$$

when $t = 0$ $\quad V_{(R_2)} = \frac{\varepsilon}{2}\left(1 - \frac{1}{3}e^{0}\right) = \frac{1}{3}\varepsilon = 400V$ } they match !!

$t = \infty$ $\quad V_{(R_2)} = \frac{\varepsilon}{2}\left(1 - \frac{1}{3}e^{-\infty}\right) = \frac{1}{2}\varepsilon = 600V$



(Extra) $\tau_c = \frac{3}{2}RC$ in this case and $\infty \sim 6RC$

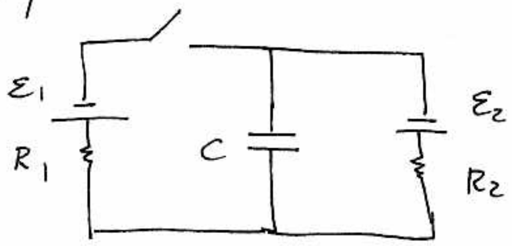
$$\text{So } \infty = 6RC = 6 \cdot \frac{3}{2}RC$$

$$= 9\,(0.73\times10^{6})(6.5\times10^{-6})$$

$$= \underline{\underline{42.705\,\text{sec}}}$$

So Physicsly, $t \geq 42.705$ sec is ∞ for this circuit.

#89



$C = 10\mu F$

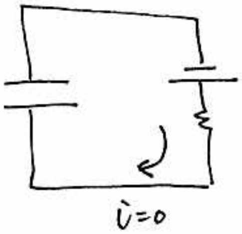
$\mathcal{E}_1 = 1V$

$\mathcal{E}_2 = 3V$

$R_1 = 0.2\Omega$

$R_2 = 0.4\Omega$

First condition

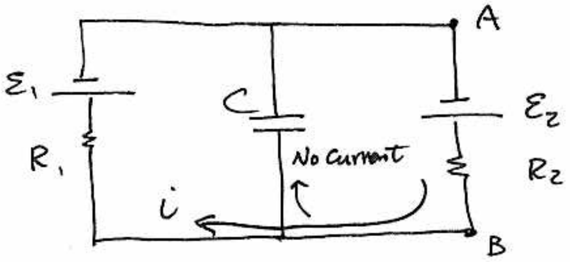


Since the cap is fully charged, $i=0$ and $V_C = \mathcal{E}$

$\therefore \quad Q_i = CV$

$= 10\mu F \cdot 3V = \underline{\underline{30\mu C}}$

Second condition



Loop 1 at A

$\mathcal{E}_2 - iR_2 - iR_1 - \mathcal{E}_1 = 0$

$\mathcal{E}_2 - \mathcal{E}_1 - i(R_1 + R_2) = 0$

$3 - 1 - i(0.2 + 0.4) = 0$

$2 - i(0.6) = 0$

$\therefore \quad i = \frac{2}{0.6} = \frac{10}{3}$ amp

$V_C = V_{AB} = \mathcal{E}_2 - iR_2$

$= 3 - \frac{10}{3} \cdot 0.4 = \underline{\frac{5}{3}V}$

$\therefore \quad Q_f = CV$

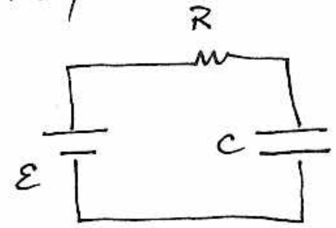
$= 10\mu F \cdot \frac{5}{3}V = \underline{\underline{\frac{50}{3}\mu C}}$

$\Delta Q = Q_f - Q_i = \frac{50}{3}\mu C - 30\mu C$

$= \frac{50}{3}\mu C - \frac{90}{3}\mu C$

$= \underline{\underline{-\frac{40}{3}\mu C}}$

#67



(a) Energy stored in the cap. when it is fully charged

$$U = \frac{1}{2} C V^2 \qquad q = CV \rightarrow C = \frac{q}{V}$$

$$= \frac{1}{2} \frac{q}{V} \cdot V^2$$

$$= \underline{\underline{\frac{1}{2} q V^2}}$$

while the charge "q" is moving to the cap., the work or energy released by the battery is

$$W = \underline{\underline{qV}}$$

Hence a half of work done by the battery is stored in the cap. (the other half? It was used up by the resistor)

(b)

$$V = V_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

(I started w/ this, but you should be able to derive this w/o any trouble)

since

$P = I^2 R$ & $W = \int P . dt$,

$W = \int I^2 R . dt$.

We need to get $I(t)$.

$$q = CV = CV_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

$$\therefore \; i(t) = \frac{dq}{dt} = \frac{d\left(CV_0\left(1 - e^{-\frac{t}{RC}}\right)\right)}{dt} = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

$$\therefore \; W = \int i^2 R \, dt$$

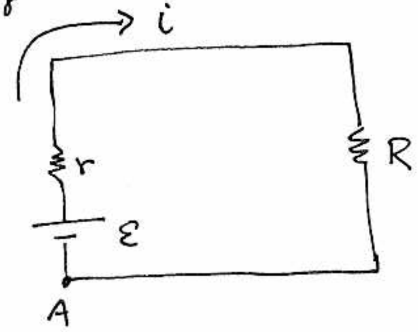
$$= \int \left(\frac{V_0}{R} e^{-\frac{t}{RC}}\right)^2 \cdot R \, dt$$

$$= \int \frac{V_0^2}{R} e^{-\frac{2t}{RC}} \cdot dt$$

$$= \frac{V_0^2}{R} \left(-\frac{RC}{2}\right) e^{-\frac{2t}{RC}} \Big|_0^{\infty}$$

$$= \frac{V_0^2 C}{2} = \frac{1}{2} C V_0^2$$ ... the same result.

#68



starting at A

$\varepsilon - ir - iR = 0$

$V = i(r+R)$

$i = \dfrac{V}{r+R}$ ——— ①

$P = iV = i^2 R$ ——— ②

② ← ①

$P = \left(\dfrac{V}{r+R}\right)^2 R$ ——— ②'

$= V^2 \cdot \dfrac{R}{(r+R)^2}$

To maximize P, set $\dfrac{dP}{dR} = 0$ (Because V & r are const. only R is a factor to decide P)

$$\frac{dP}{dR} = \frac{d\left(V^2 \cdot \frac{R}{(r+R)^2}\right)}{dR} = V^2\left(\frac{1}{(r+R)^2} - \frac{2R}{(r+R)^3}\right)$$

$$= V^2\left(\frac{(r+R) - 2R}{(r+R)^3}\right)$$

$$= V^2\left(\frac{r-R}{(r+R)^3}\right) = 0$$

$\therefore r - R = 0$ or $\underline{\underline{r = R}}$ ——— ②''

②' ← ②''

$P = \left(\dfrac{V}{r+r}\right)^2 \cdot r$

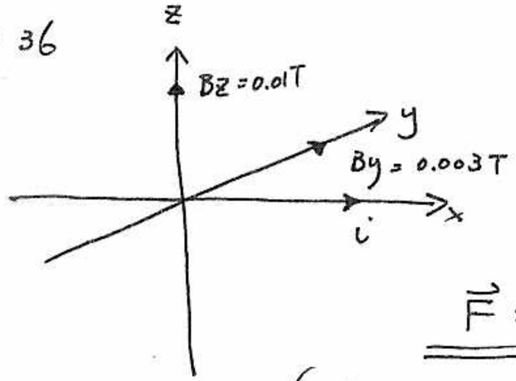
$= \left(\dfrac{V}{2r}\right)^2 \cdot r$

$= \dfrac{V^2}{4r^2} \cdot r$

$= \underline{\underline{\dfrac{V^2}{4r}}}$

Ch 28 # 36, 45, 55, 57, 82

\# 36



Use Left hand Rule

$F_x = 0$

$F_y = i\ell \times B_z = -0.5\times 0.5\times 0.01 = -0.25\times 10^{-3}N$

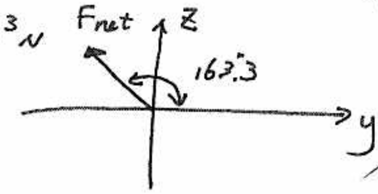
$F_z = i\ell \times B_y = 0.5\times 0.5\times 0.003 = 7.5\times 10^{-4}N$

$$\underline{\underline{\vec{F} = 0\hat{i} - 0.25\times10^{-3}N\hat{j} + 7.5\times10^{-4}N\hat{k}}}$$

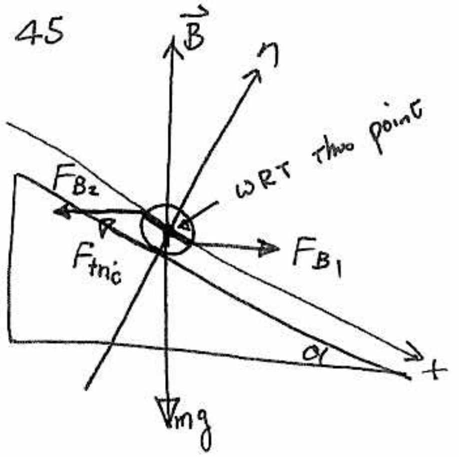
(Also,

$|F| = \sqrt{(F_x)^2+(F_y)^2+(F_z)^2} = 2.61\times10^{-3}N$

$\theta = \tan\frac{F_z}{F_y} = 163.^\circ30$)



\# 45



Linear Motion

X axis

$F_{B_1}\cos\theta + mg\sin\theta - F_{fric} - F_{B_2}\cos\theta = 0$ —— ①

y axis

$F_{B_1}\sin\theta + n - mg\cos\theta - F_{B_2}\sin\theta = 0$

$|F_{B_1}| = |F_{B_2}| = N(i\ell B)$

∴ eqn ① becomes

$mg\sin\theta - F_{fric} = 0$ —— ①'

Rotational Motion

$\vec{r}\times\vec{F}_{fric} + \vec{r}\times\vec{F}_{B_1} + \vec{r}\times\vec{F}_{B_2} = 0$

$rF_{fric} - rF_{B_1}\sin\theta - rF_{B_2}\sin\theta = 0$

$rF_{fric} - 2rF_B\sin\theta = 0$

∴ $F_{fric} = 2F_B\sin\theta$ —— ②

①' ← ②

$mg\sin\theta - 2F_B\sin\theta = 0$

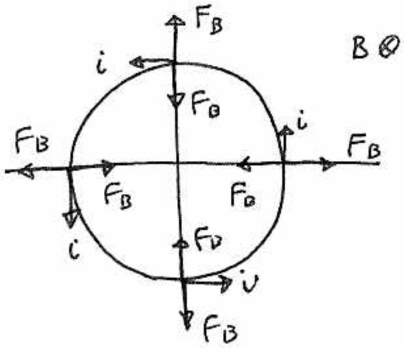
$mg\sin\theta = 2F_B\sin\theta$

$mg = 2N(i\ell B)$

∴ $i = \frac{mg}{2N\ell B} = \underline{2.4525\text{ amp}}$

55.

(a) Case 1. If B is ⊥ to the loop.

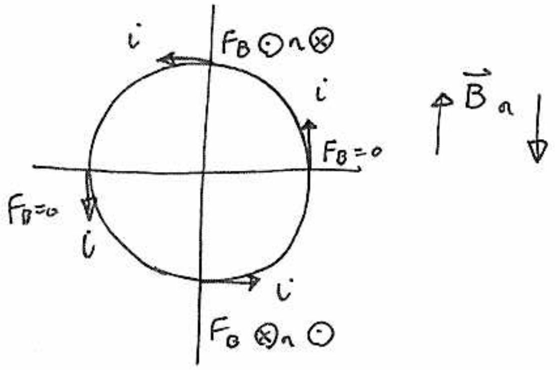


B ⊗ n ⊙

As you can see, $\vec{F}_B // \vec{r}$.

∴ $\tau = 0$.

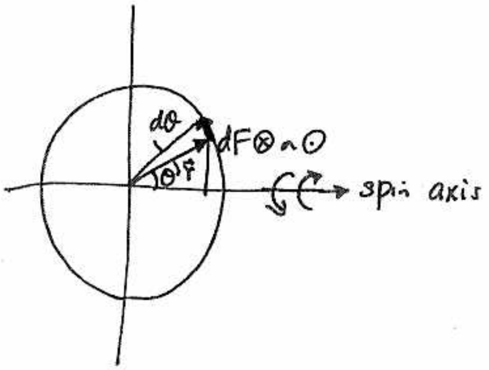
Case 2 If B is // to the loop



As you can see there is some net τ.

∴ $\underline{\underline{\vec{B} // \text{the loop}}}$

(b) We don't need a dipole eqn : you can derive it (of course!)



$$d\tau = \vec{r} \times d\vec{F}$$

$$d\vec{F} = i\, d\vec{s} \times \vec{B}$$

$$= i\, R d\theta \cdot \sin\theta \cdot B$$

$$= R\sin\theta \cdot i R d\theta \sin\theta \cdot B$$

$$= R^2 \sin^2\theta\ i\ B\ d\theta$$

$$\tau = 4 \times \int_0^{\pi/2} R^2 i B \sin^2\theta \cdot d\theta$$

$$= 4R^2 iB \int_0^{\pi/2} \sin^2\theta\, d\theta$$

$$= 4R^2 iB \int_0^{\pi/2} \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= 4R^2 iB \quad \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \Big|_0^{\pi/2}$$

$$= 4R^2 iB \quad \frac{\pi}{4}$$

$$= \pi R^2 iB$$

$$= A\, i\, B$$

$\sin^2\theta = 1 - \cos^2\theta$

Also,

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

∴ $\cos^2\theta = \cos 2\theta + \sin^2\theta$

∴ $\sin^2\theta = 1 - (\cos 2\theta + \sin^2\theta)$

$\sin^2\theta = 1 - \cos 2\theta - \sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

∴ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\therefore \tau_{Total} = NAiB$

$N = \#$ of turns $= \frac{L}{2\pi r}$

L: total length
r: radius of each circle
A: Area of a circle = πr^2

$= \frac{L}{2\pi r} \cdot (\pi r^2)\ i\ B$

$= \frac{L}{2}\ r\ i\ B$

$\therefore \tau \propto r$ For max τ, choose max $r \rightarrow N = 1$

(c) when $N = 1$

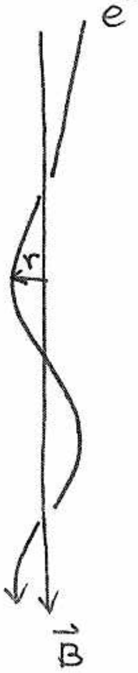
$\tau = 1 \cdot \pi r^2 \cdot iB$ $L = 2\pi r$

$= 1 \cdot \pi \left(\frac{L}{2\pi}\right)^2 iB$ $r = \frac{L}{2\pi}$

$= \frac{L^2 i B}{4\pi} = \frac{(0.25)^2 \cdot (4.51 \times 10^{-3})(5.71 \times 10^{-3})}{4\pi}$

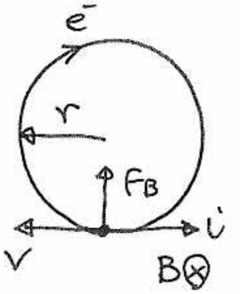
$= \underline{\underline{1.28084378 \times 10^{-7}\ N \cdot m}}$

#57



seen From the above

$m_e = 9.11 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ C
$V = 1.5 \times 10^{7}$ m/sec
$B = 1 \times 10^{-3}$ T



We need perpendicular comp. of V w.R.T. B ($V \sin 10°$)

$F_B = q \vec{V} \times \vec{B} = F_{centri} = m\frac{V^2}{r}$

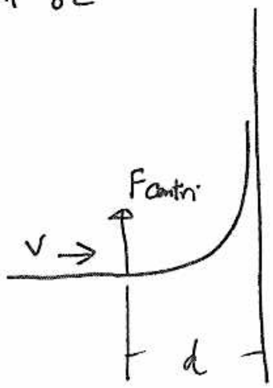
$\therefore r = \frac{m(V\sin 10°)^2}{qBV\sin 10°} = \frac{mV\sin 10°}{qB}$

P (Time needed for e^- to make 1 rev.)

$= \frac{2\pi r}{V_\perp} = \frac{2\pi\left(\frac{mV\sin 10°}{qB}\right)}{V \sin 10°} = \frac{2\pi m}{qB}$

$\therefore d = (V\cos 10°) \cdot P = \underline{\underline{0.5284707815\,m}}$

\# 82



$$F_{centri} = m\frac{V^2}{r} = m\frac{V^2}{d} \text{ (in this case)}$$

$$= F_B = evB$$

$$\therefore \; m\frac{V^2}{d} = evB$$

$$B = \frac{mV}{ed} \quad \text{——————} \; ①$$

Also

$$KE = \frac{1}{2}mV^2 = K$$

$$V = \sqrt{\frac{2K}{m}} \quad \text{——————} \; ②$$

① ← ②

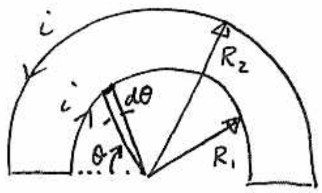
$$B = \frac{m\sqrt{\frac{2K}{m}}}{ed}$$

$$= \sqrt{\frac{2Km}{e^2d^2}}$$

As long as B is stronger than this, the particle will not hit.

Ch 29. # 4, 5, 17, 19, 25, 44, 50,

#4



$$dB(R_1) \underset{\otimes}{} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

$$= \frac{\mu_0 i}{4\pi}\frac{ds}{R_1^2}$$

$$= \frac{\mu_0 i}{4\pi}\frac{R.d\theta}{R_1^2} = \underline{\frac{\mu_0 i}{4\pi R_1}d\theta}$$

$$\underset{\odot}{dB(R_2)} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3} = \underline{\frac{\mu_0 i}{4\pi R_2}d\theta}$$

$$dB(R_1+R_2) = \frac{\mu_0 i}{4\pi R_1} - \frac{\mu_0 i}{4\pi R_2}d\theta$$

Because $|dB(R_1)| > |dB(R_2)|$ & opposite directions

$$= \frac{\mu_0 i}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)d\theta$$

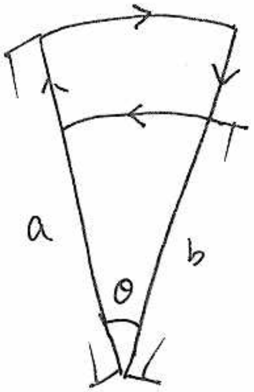
$$\therefore\ B(R_1+R_2) = \int dB = \int_0^{\pi}\frac{\mu_0 i}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)d\theta$$

$$= \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\otimes$$

$$= \frac{4\pi\times10^{-7}\cdot0.281}{4}\left(\frac{1}{0.0315} - \frac{1}{0.0780}\right)$$

$$= \underline{\underline{1.670721221\times10^{-6}\,\mathrm{T}\ \otimes}}$$

#5



Very similar to #4

$R_1 \to b$

$R_2 \to a$

limits $(0\to\pi) \to (0\to\theta)$

$$\therefore\ B(a+b) = \int_0^{\theta}\frac{\mu_0 i}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)d\theta$$

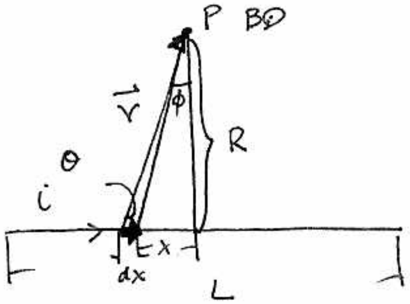
$$= \frac{\mu_0 i}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)\theta\ \odot$$

Do not forget to use "Radians".

$$= \frac{4\pi\times10^{-7}\cdot0.411}{4\pi}\left(\frac{1}{0.107} - \frac{1}{0.135}\right)\cdot\left(74^\circ\cdot\frac{\pi}{180}\right)$$

$$= \underline{\underline{1.028943183\times10^{-7}\,\mathrm{T}\ \odot}}$$

#17



$$dB = \frac{\mu_0 i}{4\pi}\,\frac{d\vec{x}\times\vec{r}}{r^3}$$

$$= \frac{\mu_0 i}{4\pi}\,\frac{dx\cdot r}{r^3}\sin\theta \qquad r = (x^2+R^2)^{1/2}$$

$$= \frac{\mu_0 i}{4\pi}\,\frac{R\,dx}{(x^2+R^2)^{3/2}} \qquad \sin\theta = \frac{R}{r} = \frac{R}{(x^2+R^2)^{1/2}}$$

Let $\tan\phi = \frac{x}{R}$

$$x = R\tan\phi$$

$$dx = R\sec^2\phi\,d\phi$$

$$= \frac{\mu_0 i}{4\pi}\,\frac{R\,(R\sec^2\phi\,d\phi)}{(R^2\tan^2\phi + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{4\pi}\,\frac{R^2}{R^3}\,\frac{\sec^2\phi}{\sec^3\phi}\,d\phi$$

$$= \frac{\mu_0 i}{4\pi R}\cdot\cos\phi\,d\phi$$

$$\therefore\quad B = \int dB = \int \frac{\mu_0 i}{4\pi R}\cos\phi\,d\phi$$

$$= \frac{\mu_0 i}{4\pi R}\sin\phi\,\Big|$$

$$= \frac{\mu_0 i}{4\pi R}\,\frac{x}{(x^2+R^2)^{1/2}}\Bigg|_{-\frac{1}{2}L}^{+\frac{1}{2}L} = 2\,\frac{\mu_0 i}{4\pi R}\,\frac{x}{(x^2+R^2)^{1/2}}\Bigg|_{0}^{\frac{1}{2}L}$$

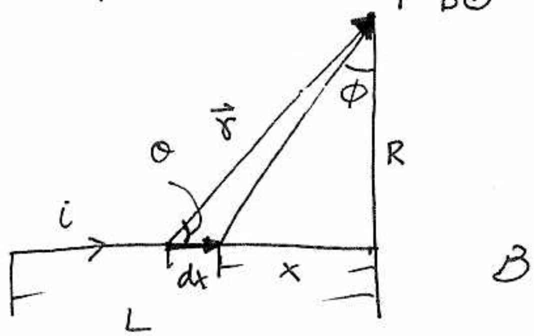
$$= 2\,\frac{\mu_0 i}{4\pi R}\left[\frac{\frac{1}{2}L}{((\frac{1}{2}L)^2+R^2)^{1/2}} - 0\right]$$

$$= \frac{\mu_0 i}{4\pi R}\,\frac{L}{(\frac{1}{4}L^2+R^2)^{1/2}} = \frac{\mu_0 i}{2\pi R}\,\frac{L}{(L^2+4R^2)^{1/2}}$$

$$= \frac{4\pi\times10^{-7}\cdot(58.2\times10^{-3})}{2\pi\,(0.131)}\cdot\frac{(0.18)}{((0.18)^2+4\,(0.131)^2)^{1/2}}$$

$$= \underline{\underline{5.031516772\times10^{-8}\,\mathrm{T} \qquad \odot}}$$

#19



The set up is exactly the same as #17. I will skip dB part since it is shown in #17. The only difference between these problems is the limits.

$$B = \int dB = \int \frac{\mu_0 i}{4\pi R} \cos\phi \, d\phi$$

$$= \frac{\mu_0 i}{4\pi R} \sin\phi \Big| = \frac{\mu_0 i}{4\pi R} \frac{x}{(x^2+R^2)^{1/2}} \Big|_{-L}^{0}$$

$$= 0 - \frac{\mu_0 i}{4\pi R} \frac{-L}{(L^2+R^2)^{1/2}}$$

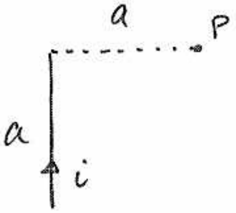
$$= \frac{\mu_0 i}{4\pi R} \frac{L}{(L^2+R^2)^{1/2}}$$

$$= \frac{4\pi\times10^{-7}\cdot(0.693)}{4\pi\,(0.251)} \cdot \frac{(0.136)}{((0.136)^2+(0.251)^2)^{1/2}}$$

$$= \underline{\underline{1.315308451\times10^{-7}\,\mathrm{T}}} \odot$$

#25

If you can do #17 & 19, this problem is just a combination of these. You should be able to start from the very beginning (dB part – but that is shown in #17). I will modify #19.

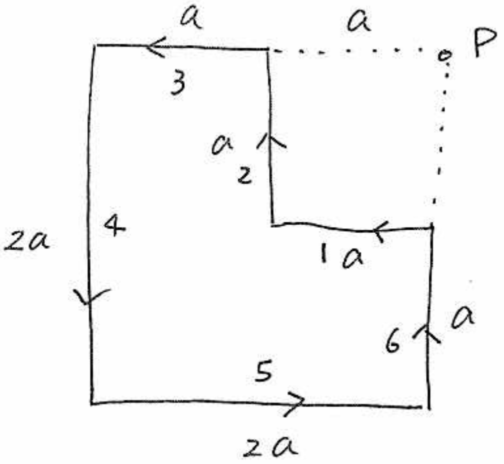


$$B = \frac{\mu_0 i}{4\pi R} \frac{L}{(L^2+R^2)^{1/2}} \quad \begin{cases} L = a \\ R = a \end{cases}$$

$$= \frac{\mu_0 i}{4\pi a} \frac{a}{(a^2+a^2)^{1/2}}$$

$$= \frac{\mu_0 i}{4\pi} \frac{1}{(2a^2)^{1/2}}$$

$$= \frac{\mu_0 i}{4\pi} \frac{1}{\sqrt{2}\,a} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}\,\mu_0 i}{8\pi a}}}$$



$$B_1 = B_2 = \frac{\sqrt{2}\,\mu_0 i}{8\pi a} \quad (\text{down at } P)$$

$$B_3 = B_6 = 0$$

$$B_4 = B_5 = \frac{\sqrt{2}\,\mu_0 i}{8\pi\,(2a)} \quad (\text{up})$$

$$\therefore \sum B_i = \frac{\sqrt{2}\,\mu_0 i}{8\pi a} \times 2 - \frac{\sqrt{2}\,\mu_0 i}{8\pi\,(2a)} \times 2$$

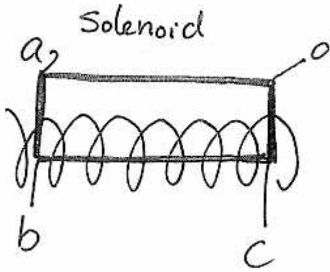
$$= \frac{\sqrt{2}\,\mu_0 i}{8\pi a} \quad (\text{down})$$

$$= \frac{\sqrt{2} \cdot 4\pi \times 10^{-7} \cdot (13)}{8\pi \quad (0.047)}$$

$$= \underline{\underline{1.95582726\overline{7} \times 10^{-5}\,\text{T} \otimes}}$$

As you can see, these problems are very similar. The point is that the application of the B-S law is very limited. Make sure you can solve these questions w/o any problem — practice!

#44



Ampere's Law

$$\oint B \cdot dS = \mu_0 i$$

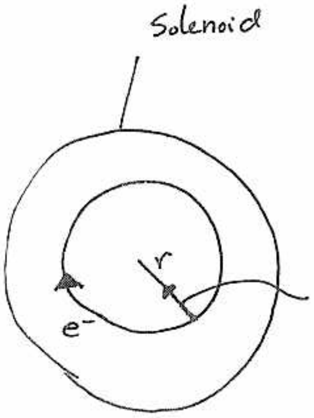
$$= \int_a^b B \cdot dS + \int_b^c B \cdot dS + \int_c^d B \cdot dS + \int_d^a B \cdot dS$$

$$= 0 + \int_b^c B \cdot dS + 0 + 0$$

$$= B \cdot \overline{bc} = \mu_0 i_{\text{inside}}$$

$$B \cdot \overline{bc} = \mu_0 i\, n\, \overline{bc} \qquad n : \#\ \text{turns}/\text{unit length}$$

$$\therefore \quad B = \mu_0 i n \quad \text{———} ①$$



$$F_{\text{centri}} = F_B$$

$$m_e \frac{V^2}{r} = e V \times B \quad \text{———} ②$$

② ⟵ ①

$$m_e \frac{V^2}{r} = e V (\mu_0 i n)$$

$$i = \frac{m_e V}{\mu_0 e n r}$$

$V = 0.046\ C$ $(C = 3.0 \times 10^8 \text{ m/sec})$

$e = 1.6 \times 10^{-19}\ C$

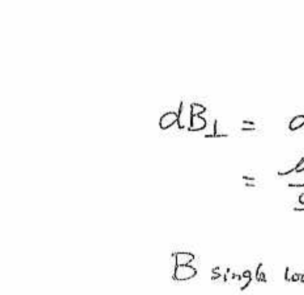
$n = 10000$ Turns/m

$r = 2.3$ cm $= 0.023$ m

$m_e = 9.11 \times 10^{-31}$ kg

$$= \underline{\underline{2.71856537 \times 10^{-1}\ \text{Amp}}}$$

50



calculate dB_z by one loop. B_{Total} is Twice as much.

$$dB = \frac{\mu_0 i}{4\pi} \frac{d\vec{s} \times r}{r^3} \qquad r^2 = (\tfrac{1}{2}R)^2 + R^2$$

$$= \frac{\mu_0 i}{4\pi} \frac{ds}{r^2} \qquad = \tfrac{5}{4} R^2$$

$$= \frac{\mu_0 i}{4\pi} \frac{dS}{\frac{5}{4} R^2} = \frac{\mu_0 i}{5\pi} \frac{ds}{R^2}$$

$$dB_\perp = dB \cos\theta$$

$$= \frac{\mu_0 i}{5\pi R^2} \cdot dS \cdot \frac{R}{r} = \frac{\mu_0 i}{5\pi R^2} \cdot dS \cdot \frac{R}{\sqrt{\frac{5}{4}} R} = \frac{2\mu_0 i}{5\sqrt{5}\, \pi R^2} \cdot dS$$

$$B_{\text{single loop}} = \oint dB_\perp = \frac{2\mu_0 i}{5\sqrt{5}\, \pi R^2} \cdot 2\pi R = \frac{4\mu_0 i}{5\sqrt{5}\, R}$$

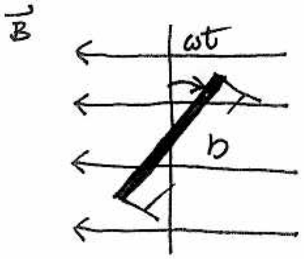
$$B_{n\text{ loops}} = \frac{4 n \mu_0 i}{5\sqrt{5}\, R}$$

$$\underline{\underline{B_{\text{Total}} = B_{n\text{ loops}} \times 2 = \frac{8 n \mu_0 i}{5\sqrt{5}\, R}}} = \frac{8 \cdot 200 \cdot 4\pi \times 10^{-7} \cdot 12.2 \times 10^{-3}}{5\sqrt{5} \cdot 0.25} = \underline{\underline{8.775960547 \times 10^{-6}\ T}}$$

Ch. 30 # 11, 24, 30, 32, 33, 39, 41, 45, 46, 53, 62, 68, 72, 89, 92

\# 11. seen from the edge

(a)



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d\int B \cdot dA}{dt}$$

$$= -\frac{d[N\,\overbrace{(B \cdot (ab)\cos\theta)}^{\text{Mag. flux per turn}}]}{dt}$$

$$= -\frac{d\,[NBab\cos(\omega t)]}{dt}$$

$$= \omega NBab \sin(\omega t)$$

since $\omega = 2\pi\nu$

$$= \underbrace{2\pi\nu NBab}\sin(2\pi\nu t)$$

$$= \underline{\underline{\mathcal{E}_0 \sin(2\pi\nu t)}}$$

(b)

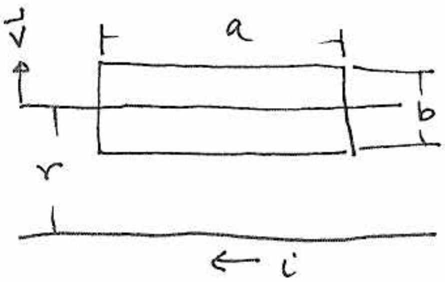
$$\mathcal{E}_0 = 2\pi\nu NB(ab) = 150\,V$$

$$60\ \text{rev/sec} = 60\ Hz = 60\,\frac{\text{rev}}{\text{sec}} \cdot \frac{2\pi\ \text{rad}}{\text{rev}} = 120\pi\ \text{rad/sec}$$

$$150\,V = \omega NBab$$

$$Nab = \frac{150V}{\omega B} = \frac{150\,V}{120\pi\ \text{rad/sec} \cdot 0.5\,T} = \underline{\underline{0.795774715\ m^2}}$$

\# 24



The direction of the induced i is clockwise

a = 2.2 cm = 0.022 m
b = 0.8 cm = 0.008 m
$R = 0.4\,m\Omega = 4\times10^{-4}\,\Omega$
i = 4.7 A
$V = 3.2\ mm/sec = 3.2\times10^{-3}\ m/sec$
r = 1.5 b

(a)
$$\Phi_B = \int B \cdot dA$$

$$= \int \frac{\mu_0 i}{2\pi\gamma}\, a \cdot d\gamma$$

We did this in the lecture. If you are not sure how this is done, see your lecture note

$$= \frac{\mu_0 i a}{2\pi} \ln\gamma \Big|_{r-b/2}^{r+b/2}$$

$$= \frac{\mu_0 i a}{2\pi} \ln\frac{r+\frac{b}{2}}{r-\frac{b}{2}}$$

$$= \frac{4\pi\times10^{-7} \cdot 4.7 \cdot 0.022}{2\pi} \ln\left[\frac{1.5(0.008)+\frac{0.008}{2}}{1.5(0.008)-\frac{0.008}{2}}\right]$$

$$= \underline{\underline{0.143342836 \times 10^{-8}\ Wb}}$$

Make sure to use MSK (meter-second-Kilogram)!!

(b)

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d\left[\frac{\mu_0 i a}{2\pi}\ln\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right]}{dt}$$

$$= -\frac{\mu_0 i a}{2\pi}\left[\frac{1}{r+\frac{b}{2}}\cdot\frac{dr}{dt} - \frac{1}{r-\frac{b}{2}}\cdot\frac{dr}{dt}\right]$$

$$= -\frac{\mu_0 i a}{2\pi}\left[\frac{1}{r+\frac{b}{2}} - \frac{1}{r-\frac{b}{2}}\right]\cancel{\frac{dr}{dt}}^{v}$$

$$= -\frac{\mu_0 i a v}{2\pi}\left[\frac{(r-\frac{b}{2})-(r+\frac{b}{2})}{(r+\frac{b}{2})(r-\frac{b}{2})}\right]$$

$$= -\frac{\mu_0 i a v}{2\pi}\frac{-b}{r^2-(\frac{b}{2})^2}$$

$$= \frac{\mu_0 i a b v}{2\pi}\frac{1}{(r^2-\frac{b^2}{4})}$$

$$= \frac{\mu_0 i a b v}{2\pi}\frac{1}{\frac{1}{4}(4r^2-b^2)}$$

$$= \frac{2\mu_0 i a b v}{\pi}\cdot\frac{1}{(4r^2-b^2)}$$

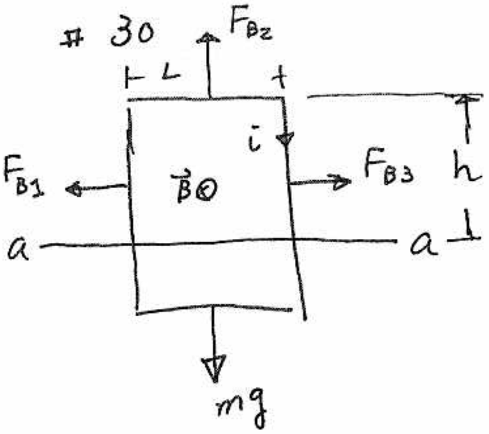
$$= \frac{2\cdot 4\pi\times10^{-7}\cdot(4.7)(0.022)(0.008)(3.2\times10^{-3})}{\pi}\quad\frac{1}{(4(1.5b)^2-b^2)}$$

$$= 8\times10^{-7}(4.7)(0.022)(0.008)(3.2\times10^{-3})\cdot\frac{1}{8(0.008)^2}$$

$$= 4.136\times10^{-9}\,V$$

$$\therefore\ i = \frac{\mathcal{E}}{R} = \frac{4.136\times10^{-9}\,V}{4\times10^{-4}\,\Omega} = \underline{\underline{1.034\times10^{-5}\ \text{amp}}}$$

30



x comp

$\vec{F}_{B1}+\vec{F}_{B3}=0$

y comp

$F_{B2}-mg=ma$

Also

$F_{B2}=i_{ind}\cdot L\ B$ & $a=0$ for terminal speed.

$\therefore\ F_{B2}-mg=\cancel{ma}^{\to 0}$

$i_{ind}LB=mg$ ——— ①

Induced "i"

$$i=\frac{\varepsilon}{R}=\frac{1}{R}\frac{d\Phi_B}{dt}=\frac{1}{R}\frac{-d\int B\cdot dA}{dt}=\frac{-1}{R}\frac{d(BLh)}{dt}$$

$$=-\frac{1}{R}\ BL\ \frac{dh}{dt}\to(-V)$$ (h is getting shorter)

$$=\frac{BLV}{R}$$ ——— ②

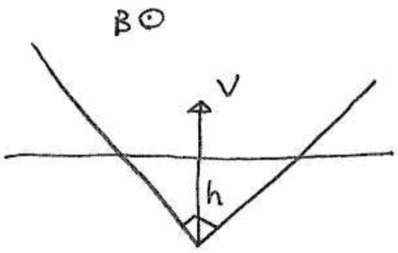
① ← ②

$$\frac{BLV}{R}\cdot L\ B=mg$$

$$\frac{B^2L^2V}{R}=mg$$

$$\therefore\ V=\underline{\underline{\frac{mgR}{B^2L^2}}}$$

32



at $t=0$, $V=5.2$ m/sec w/ $a=0$ m/sec²

$B=0.35\ T$

For this triangle base is "2h" & height is "h"

(a) at $t=3$ sec, $h=5.2$ m/sec $\cdot$ 3 sec $=15.6$ m

$\Phi_B=\int B\cdot dA$

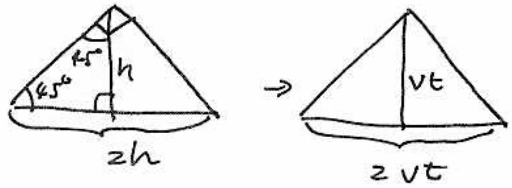
$=B\cdot\text{Area}=B\frac{1}{2}(2h\cdot h)=Bh^2$

$=0.35(15.6)^2=\underline{\underline{85.176\ wb}}$

(b) $$\mathcal{E}=\frac{d\Phi_B}{dt}=\frac{d(Bh^2)}{dt}=\left(B\,2h\,\frac{dh}{dt}\right)$$

$$=0.35\cdot2\cdot(15.6)\cdot5.2=\underline{\underline{56.784\ V}}$$

(c) $h = vt$



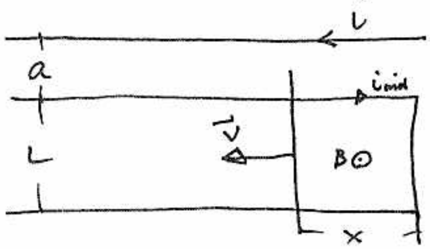
$$\text{Area} = \frac{1}{2}(2vt)(vt)$$
$$= (vt)^2$$

$$\therefore \ \Phi_B = B\cdot(vt)^2$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(Bv^2t^2)}{dt} = BV^2\cdot 2t$$
$$= 2BV^2\cdot t = at^b$$

$$\boxed{\begin{aligned}\therefore \ a &= 2BV^2 \\ b &= 1\end{aligned}}$$

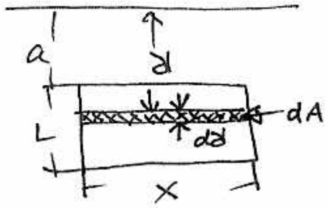
\# 33



"i_{ind} is clockwise"

$L = 0.1\,m$
$V = 5\,m/sec$
$R = 0.4\,\Omega$
$i = 100\,A$ (I wanna see the wire ···N,t a cable !)
$a = 0.01\,m$

(a) $$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d\int B\cdot dA}{dt} = \frac{d\int \frac{\mu_0 i}{2\pi \partial}\cdot d\partial \cdot x}{dt}$$



$$= \frac{d\left[\frac{\mu_0 i x}{2\pi}\ln \partial \Big|_a^{a+L}\right]}{dt}$$

$$= \frac{d\left[\frac{\mu_0 i x}{2\pi}\ln\frac{a+L}{a}\right]}{dt}$$

$$= \frac{\mu_0 i}{2\pi}\ln\left|\frac{a+L}{a}\right|\cdot \cancel{\frac{dx}{dt}}^{\,V}$$

$$= \frac{\mu_0 i}{2\pi}\ln\frac{a+L}{a}\cdot V$$

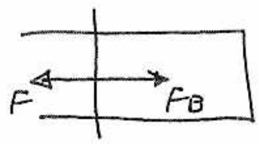
$$= \frac{4\pi\times 10^{-7}\cdot 100}{2\pi}\cdot \ln\frac{0.01+0.1}{0.01}\cdot 5$$

$$= \underline{\underline{2.397895273\times 10^{-4}\,V}}$$

(b) $$i = \frac{\mathcal{E}_{ind}}{R} = \frac{2.397\cdots\times 10^{-4}\,V}{0.4\,\Omega} = \underline{\underline{5.994738182\times 10^{-4}\,amp}}$$

(c) $$P = i^2R = \underline{\underline{1.437475435\times 10^{-7}\,\text{Watt}}}$$

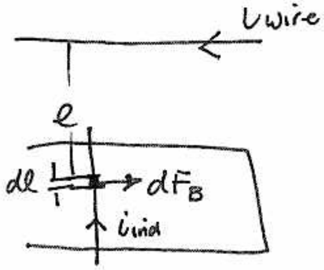
(d)



Since V is const, $a = 0$.

$$\therefore \quad F - F_B = 0$$

$$F = F_B$$



$$dF_B = i_{ind}\, dl\, B_{\text{by the wire at the loop}}$$

$$= i_{ind}\, dl\, \frac{\mu_0 i_{wire}}{2\pi l}$$

Make sure you know which "i" is which. This is a typical beginner's mistake

$$\therefore F_B = \int dF_B = \int i_{ind} \frac{\mu_0 i_{wire}}{2\pi l}\, dl$$

$$= \frac{\mu_0 i_{wire}}{2\pi} i_{ind} \,.\, \ln l \Big|_a^{a+L}$$

$$= \frac{\mu_0 i_{wire}\, i_{ind}}{2\pi} \ln \frac{a+L}{a}$$

$$= \frac{4\pi \times 10^{-7} . (100)(5.9947\ldots \times 10^{-4})}{2\pi} \ln\left(\frac{0.01+0.1}{0.01}\right)$$

$$= \underline{\underline{2.87495087 \times 10^{-8}\, N}}$$

(e) From Physics 230.

$$P = \frac{dW}{dt} = \frac{d \int F \cdot dr}{dt}$$

$$= \frac{d(Fr)}{dt} \quad (\text{for const. } F)$$

$$= F \overset{V}{\cancel{\frac{dr}{dt}}}$$

$$= (2.8749\cdots \times 10^{-8} N)(5\ m/sec)$$

$$= \underline{1.437475435 \times 10^{-7}}\ \text{watt}$$

(C) & (e) must agree because the thermal energy has to be provided by some outside source. Energy put in (from outside) = Energy spent (as thermal energy via R)

#39

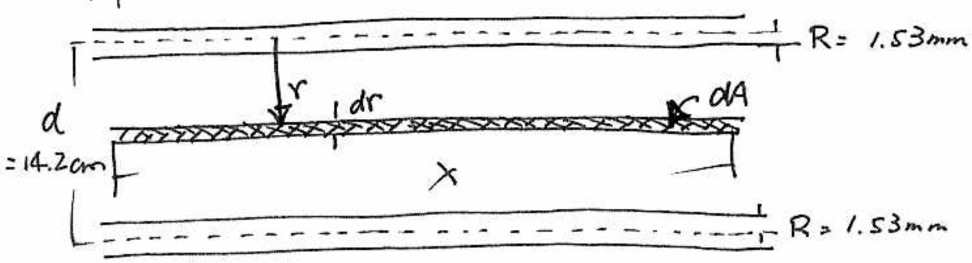
$$L\frac{di}{dt} = \frac{N\, d\overbrace{\int B\cdot dA}^{\text{per turn}}}{dt}$$

$$\int L\, di = \int N\, d\,\Phi_B$$

$$Li = N\Phi_B$$

$$\Phi_B = \frac{Li}{N} = \frac{8\times10^{-3}\cdot 5\times10^{-3}}{400} = \frac{40\times10^{-6}}{400} = \underline{\underline{1\times10^{-7}\ \text{Wb}}}$$

#41



$$\Phi_B = \int B\cdot dA$$

$$= \int_R^{d-R} \frac{\mu_0 i}{2\pi r}\cdot x\, dr$$

$$= \frac{\mu_0 i x}{2\pi} \ln r \Big|_R^{d-R}$$

from both wires

$$= \frac{\mu_0 i x}{2\pi} \ln\frac{d-R}{R} \Rightarrow \Phi_{B,\text{Total}} = \overset{\uparrow}{2}\times\Phi_B = \frac{\mu_0 i x}{\pi}\ln\frac{d-R}{R}$$

$$L\frac{di}{dt} = \frac{d\Phi_B}{dt}$$

$$\int L\, di = \int d\Phi_B$$

$$Li = \frac{\mu_0 i x}{\pi}\ln\frac{d-R}{R}$$

$$L = \frac{\mu_0 x}{\pi}\ln\frac{d-R}{R}$$

$$\frac{L}{x} = \frac{\mu_0}{\pi}\ln\frac{d-R}{R}$$

$$= \frac{4\pi\times10^{-7}}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = \underline{\underline{1.807890483\times10^{-6}\ \text{H/m}}}$$

\# 45

$$V_1 + V_2 = V_T$$

(a)

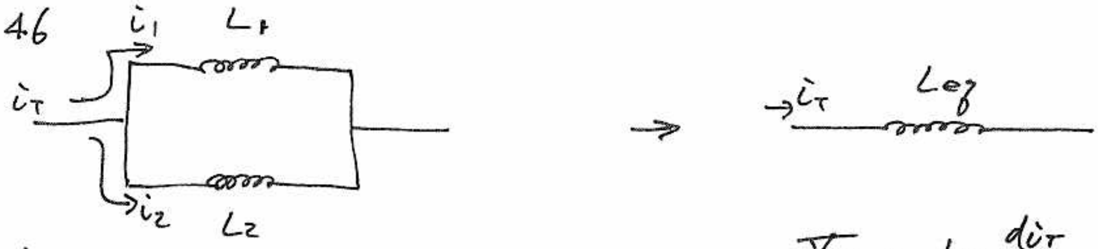
$$L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = L_{eq} \frac{di}{dt} \quad \left(\frac{di}{dt} \text{ is the same for all} \right)$$

$$\therefore \underline{\underline{L_1 + L_2 = L_{eq}}}$$

(b)

$$\underline{\underline{L_{eq} = \Sigma L_i}}$$

\# 46



(a)

$$V_1 = L_1 \frac{di_1}{dt} \Rightarrow \frac{di_1}{dt} = \frac{V_1}{d_1} \quad — ①$$

$$V_2 = L_2 \frac{di_2}{dt} \Rightarrow \frac{di_2}{dt} = \frac{V_2}{L_2} \quad — ②$$

$$V_{eq} = L_{eq} \frac{di_T}{dt}$$

$$= L_{eq} \frac{d(i_1 + i_2)}{dt}$$

$$= L_{eq} \left(\frac{di_1}{dt} + \frac{di_2}{dt} \right) \quad —— ③$$

③ ⇐ ① & ②

$$V_{eq} = L_{eq} \left(\frac{V_1}{L_1} + \frac{V_2}{L_2} \right)$$

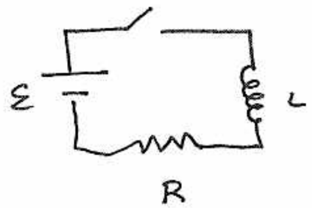
$$\frac{V_{eq}}{L_{eq}} = \frac{V_1}{L_1} + \frac{V_2}{L_2} \qquad \text{However } V_1 = V_2 = V_{eq}$$

$$\therefore \underline{\underline{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}}$$

(b)

$$\underline{\underline{L_{eq} = \frac{1}{\Sigma \frac{1}{L_i}}}}$$

#53



$\mathcal{E} = 14\,V$

$L = 6.3\times10^{-6}\,H$

$R = 1.2\times10^{3}\,\Omega$

<u>Loop</u>

$$\mathcal{E} - L\frac{di}{dt} - iR = 0$$

$$\mathcal{E} - iR = L\frac{di}{dt}$$

$$i - \frac{\mathcal{E}}{R} = -\frac{L}{R}\frac{di}{dt}$$

$$\frac{i - \frac{\mathcal{E}}{R}}{-L/R} = \frac{di}{dt}$$

$$\int \frac{dt}{-L/R} = \int \frac{di}{i - \frac{\mathcal{E}}{R}}$$

$$e^{\frac{t}{-L/R} + \gamma} = e^{\ln\left(i - \frac{\mathcal{E}}{R}\right)}$$

$$e^{-\frac{t}{L/R}} \cdot \gamma' = i - \frac{\mathcal{E}}{R}$$

at $t = 0$, $i = 0$

$$\therefore \gamma' = -\frac{\mathcal{E}}{R}$$

$$\therefore e^{-\frac{t}{L/R}} \cdot \left(-\frac{\mathcal{E}}{R}\right) = i - \frac{\mathcal{E}}{R}$$

$$\underline{\therefore i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)}$$

(a) $i = 0.8\, i_{max} = 0.8\frac{\mathcal{E}}{R}$

$$0.8\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

$$0.2 = e^{-\frac{t}{L/R}}$$

$$\ln 0.2 = -\frac{t}{L/R}$$

$$\therefore t = -\frac{L}{R}\ln 0.2$$

$$= -\frac{6.3\times10^{-6}}{1.2\times10^{3}}\ln 0.2$$

$$\underline{\underline{= 8.44954904\times10^{-9}\ \text{sec}}}$$

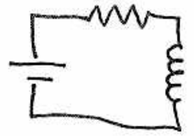
(b) at $t = 1\tau_L$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1}\right)$$

$$= \frac{14}{1.2\times10^{3}}\left(1 - e^{-1}\right)$$

$$\underline{\underline{= 7.374739853\times10^{-3}\ \text{amp}}}$$

#62



$\mathcal{E} = 10\,V$
$R = 6.7\,\Omega$
$L = 5.5H$

From # 53,

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

(a) $W = q\mathcal{E}$ (q is the total charge left from the battey between $0-2$ sec)

$$= \mathcal{E}\int dq \qquad \left(i = \frac{dq}{dt}\right)$$

$$= \mathcal{E}\int i\,dt$$

$$= \mathcal{E}\int \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)\cdot dt$$

$$= \frac{\mathcal{E}^2}{R}\left(t + \frac{L}{R}e^{-\frac{t}{L/R}}\right)\Big|_0^2$$

$$= \frac{10^2}{6.7}\left[\left(2 + \frac{5.5}{6.7}e^{-\frac{2}{5.5/6.7}}\right) - \left(0 + \frac{5.5}{6.7}e^0\right)\right]$$

$$= \underline{\underline{18.67037441\,J}}$$

(b) $E_B = \frac{1}{2}Li^2$

$$= \frac{1}{2}\,5.5\left(\frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)\right)^2 \quad \text{at } t = 2\text{ sec}$$

$$= \underline{\underline{5.10116536\,J}}$$

(c) $P = i^2R$

$$= \left[\frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{L/R}}\right)\right]^2 R$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - e^{-\frac{t}{L/R}}\right)^2$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - 2e^{-\frac{t}{L/R}} + e^{-\frac{2t}{L/R}}\right)$$

$$W = \int P\cdot dt = \int \frac{\mathcal{E}^2}{R}\left(1 - 2e^{-\frac{t}{L/R}} + e^{-\frac{2t}{L/R}}\right)dt$$

$$= \frac{\mathcal{E}^2}{R}\left(t + \frac{2L}{R}e^{-\frac{t}{L/R}} - \frac{L}{2R}e^{-\frac{t}{L/R}}\right)\Big|_0^2$$

$$= \frac{10^2}{6.7}\left[\left(2 + \frac{2(5.5)}{6.7}e^{-\frac{2}{5.5/6.7}} - \frac{5.5}{2(6.7)}e^{-\frac{2}{5.5/6.7}}\right) - \left(0 + \frac{2(5.5)}{6.7}e^0 - \frac{5.5}{2(6.7)}e^0\right)\right]$$

$$\approx \underline{\underline{13.50187263\,J}}$$

check:

(a) = (b) + (c)

Energy provided by the battey = Magnetic energy + thermal energy wasted by R

#68

$$\mathcal{E} = M \frac{di}{dt}$$

$$30\times10^{3}\,V = M \frac{6A}{2.5\times10^{-3}\,sec}$$

$$M = \frac{30\times10^{3}\,V \cdot 2.5\times10^{-3}\,sec}{6\,A}$$

$$= \underline{\underline{12.5\,H}}$$

#72

(a) $B_{sys} = \mu_0 i n$ (You should be able to derive this)

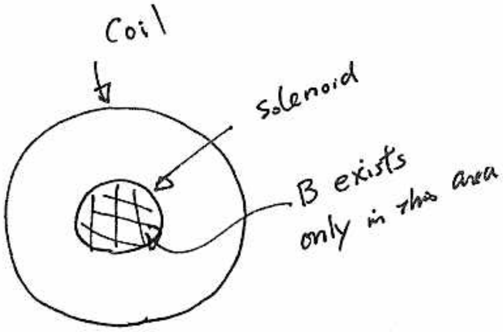
$$\Phi_B = \int B \cdot dA = \mu_0 i n \cdot \pi R^2$$ (per turn of C)

$$\therefore \Phi_{B\,Total} = N \mu_0 i n \pi R^2$$

$$M \frac{di}{dt} = \frac{d\Phi_{B\,Total}}{dt}$$

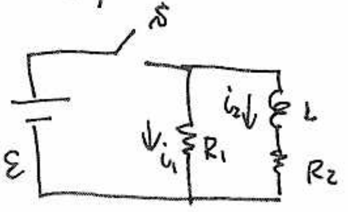
$$M i = N \mu_0 i n \pi R^2$$

$$\therefore \underline{\underline{M = N \mu_0 n \pi R^2}}$$



(b) B created by S is uniform and it exists only inside the S. So even if the shape of C changes, Φ_B will not change as long as S is inside of C.

#89



$\mathcal{E} = 10V$
$R_1 = 5\Omega$
$R_2 = 10\Omega$
$L = 5.0H$

(a) $i_1 = \frac{\mathcal{E}}{R} = \frac{10}{5} = \underline{\underline{2\,amps}}$

(b) at $t=0$, $L\frac{di}{dt} = 10V \rightarrow \underline{\underline{i_2 = 0\,amp}}$

(c) $i_T = i_1 + i_2 = 2+0 = \underline{\underline{2\,amps}}$

(d) $V_2 = i_2 R_2 = \underline{\underline{0V}}$

(e) $V_L = L\frac{di}{dt} = \underline{\underline{10V}}$ $\left(\mathcal{E} - L\frac{di_2}{dt} - i_2 \cancel{R_2}^{0} = 0 \right.$

(f) $\frac{di_2}{dt} = \frac{10V}{L} = \frac{10V}{5H} = \underline{\underline{2\,amp/sec}}$

(g) at $t = \infty$

$$i_1 = \frac{\varepsilon}{R_1} = \frac{10V}{5\Omega} = \underline{\underline{2 \text{ amp}}}$$

h) $$i_2 = \frac{\varepsilon}{R_2} = \frac{10V}{10\Omega} = \underline{\underline{1 \text{ amp}}}$$

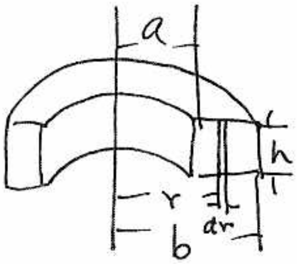
(i) $$i_T = i_1 + i_2 = 2 + 1 = \underline{\underline{3 \text{ amps}}}$$

(j) $$V_2 = i_2 R_2 = \underline{\underline{10 V}}$$

(k) $$V_L = \underline{\underline{0 V}}$$

(l) $$\frac{di_2}{dt} = \frac{0V}{L} = \underline{\underline{0 \text{ amp/sec}}}$$

#92



$$\oint B\cdot ds = \mu_0 i_{enc}$$

$$B\cdot 2\pi r = N_1 \mu_0 i$$

$$\therefore B = \frac{N_1 \mu_0 i}{2\pi r}$$

$$\Phi_{B_T} = N_2 \overbrace{\int B\cdot dA}^{\Phi_B \text{ per turn}}$$

$$= N_2 \int \frac{N_1 \mu_0 i}{2\pi r} h\cdot dr$$

$$= \frac{N_1 N_2 \mu_0 i h}{2\pi} \cdot \ln r \Big|_a^b$$

$$= \frac{N_1 N_2 \mu_0 i h}{2\pi} \ln\frac{b}{a}$$

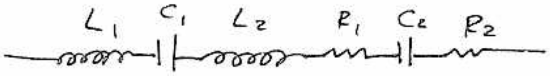
$$M\frac{di}{dt} = \frac{d\Phi_B}{dt}$$

$$Mi = \frac{N_1 N_2 \mu_0 i h}{2\pi} \ln\frac{b}{a}$$

$$\therefore \underline{\underline{M = \frac{N_1 N_2 \mu_0 h}{2\pi} \ln\frac{b}{a}}}$$

Ch. 31 # 8, 13, 44, 46, 49, 56. 57. 59, 98

8



Loop law

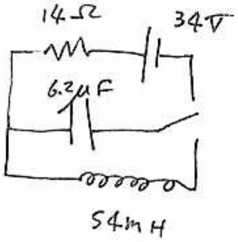
$$+L_1\frac{di}{dt}+\frac{q_1}{C_1}+iR_1+\frac{q_2}{C_2}+iR_2+\dots$$

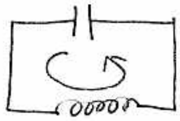
$q_1=q_2=\dots$ for series

$$(L_1+L_2+\dots)\frac{di}{dt}+\left(\frac{q_1}{C_1}+\frac{q_2}{C_2}+\dots\right)+i(R_1+R_2+\dots)$$

$$L_{eq}\frac{di}{dt}+\frac{q}{C_{eq}}+iR_{eq}$$

13.





a)

loop

$$-\frac{q}{C}-L\frac{di}{dt}=0$$

$$\frac{1}{C}q+L\frac{di}{dt}=0 \qquad i=\frac{dq}{dt}=\dot{q}$$

$$\therefore\ \frac{di}{dt}=\frac{d^2q}{dt^2}=\ddot{q}$$

$$\frac{1}{C}q+L\ddot{q}=0$$ (This is a simple harmonic oscillation!)

Let

$$q=A\cos(\omega t+\phi)$$

$$\dot{q}=-A\omega\sin(\omega t+\phi)$$

$$\ddot{q}=-A\omega^2\cos(\omega t+\phi)$$

$$\frac{1}{C}A\cos(\omega t+\phi)+L\left(-A\omega^2\cos(\omega t+\phi)\right)=0$$

$$A\cos(\omega t+\phi)\left(\frac{1}{C}-L\omega^2\right)=0$$

$$\therefore\ \frac{1}{C}-L\omega^2=0$$

$$\omega=\sqrt{\frac{1}{LC}}$$

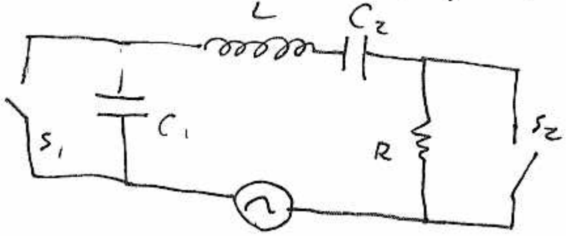
$$\nu=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=2.750599799\times10^2\,\mathrm{Hz}$$

(b) when $t=0$, q is max $(= CV = 6.2\times10^{-6}\cdot 34)$

$$q = CV = A\,\overset{1}{\cancel{\cos(\omega t+\phi)}} = 6.2\times10^{-6}\cdot 34 = 2.108\times10^{-4}\,C$$

$$\therefore \dot{q} = \underbrace{|-\omega A|}_{\text{Amp. for } \dot{q}}\,\overset{1}{\cancel{\sin(\omega t+\phi)}} = \underline{\underline{0.3643156954 \text{ amp}}}$$

44

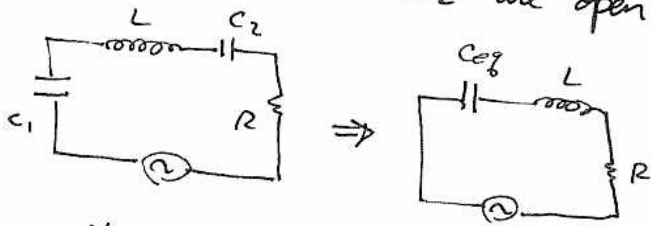


$\mathcal{E}_{max} = 12V$

$\nu = 60\text{ Hz}$

$C_1 = C_2$

Case I: when S_1 & S_2 are open, i leads $\mathcal{E}$ by 30.9°



$$C_{eq} = \frac{1}{\frac{1}{C}+\frac{1}{C}} = \frac{1}{2}C$$

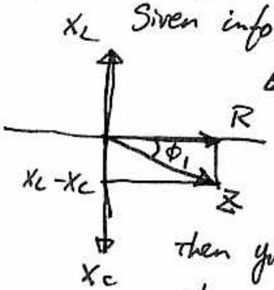
$X_L = \omega L$

$X_C = \frac{1}{\omega C_{eq}} = \frac{2}{\omega C}$

$$\phi_1 = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{\omega L - \frac{2}{\omega C}}{R} = -30.9°$$

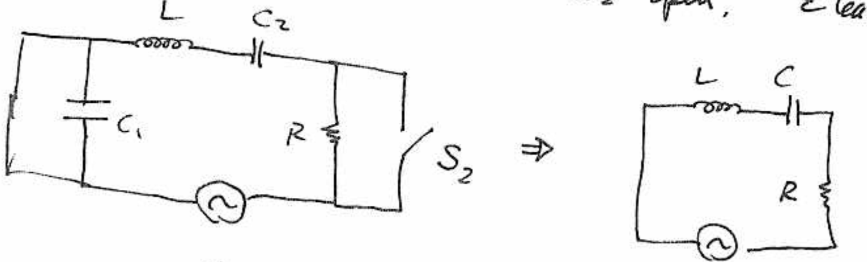
$$\boxed{\therefore\ \frac{\omega L - \frac{2}{\omega C}}{R} = \tan\phi_1 \ \text{———} \ ①}$$

You should be able to construct an impedance diagram from the given info.



Then you will see why $\phi_1 = -30.9°$

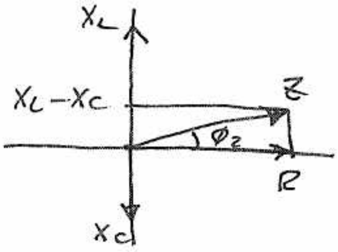
Case 2: when S_1 closed but S_2 open, $\mathcal{E}$ leads i by 15°



$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

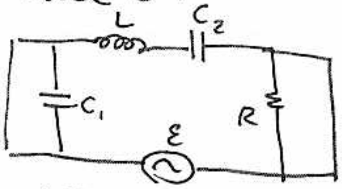
$$\phi_2 = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R} = 15.0°$$



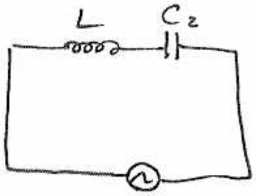
$$\boxed{\therefore\ \frac{\omega L - \frac{1}{\omega C}}{R} = \tan\phi_2 \ \text{———} \ ②}$$

$$Z = \sqrt{(X_L - X_C)^2}$$

Case 3 : Both switches closed ,
$i = 447\,mA$



$\Rightarrow$



$$= (X_L - X_C)$$

$$\frac{\varepsilon}{i} = Z = X_L - X_C = 447\,mA$$

$$\frac{\varepsilon}{i} = \omega L - \frac{1}{\omega C}$$

$$\omega L = \frac{\varepsilon}{i} + \frac{1}{\omega C}$$

$$L = \frac{\frac{\varepsilon}{i} + \frac{1}{\omega C}}{\omega} \quad \text{———} \quad ②$$

$$② \leftarrow ③$$

$$\frac{\omega\left(\frac{\frac{\varepsilon}{i} + \frac{1}{\omega C}}{\omega}\right) - \frac{1}{\omega C}}{R} = \tan 15°$$

$$\frac{\frac{\varepsilon}{i} + \frac{1}{\omega C} - \frac{1}{\omega C}}{R} = \tan 15°$$

$$\therefore \quad R = \frac{\frac{\varepsilon}{i}}{\tan 15°} = \frac{\frac{12}{447 \times 10^{-3}}}{\tan 15°} = \underline{\underline{100.1892834\,\Omega}} \quad \text{———} \quad ②'$$

$$① \leftarrow ③ \; \& \; ②'$$

$$\frac{\omega\left(\frac{\frac{\varepsilon}{i} + \frac{1}{\omega C}}{\omega}\right) - \frac{2}{\omega C}}{R} = \tan(-30.°9)$$

$$\frac{\frac{\varepsilon}{i} + \frac{1}{\omega C} - \frac{2}{\omega C}}{R} = \tan(-30.°9)$$

$$\frac{\frac{\varepsilon}{i} - \frac{1}{\omega C}}{R} = \tan(-30.°9)$$

$$\frac{\varepsilon}{i} - \frac{1}{\omega C} = R\tan(-30.°9) \Rightarrow \quad C = \frac{1}{\omega\left(\frac{\varepsilon}{i} - R\tan(-30.°9)\right)} = \frac{1}{2\pi(60)\left(\frac{12}{447 \times 10^{-3}} - R\tan(-30.°9)\right)}$$

$$= \underline{\underline{30.55695886\,\mu F}} \quad \text{———} \quad ①'$$

$$③ \leftarrow ①'$$

$$L = \frac{\frac{\varepsilon}{i} + \frac{1}{\omega C}}{\omega} = \frac{\frac{12}{447 \times 10^{-3}} + \frac{1}{2\pi \nu C}}{2\pi \nu} = \underline{\underline{301.474806\,mH}}$$

#46

$\mathcal{E}_m = 220 V$

$\nu = 400 Hz$

$R = 220 \Omega$

$L = 150 \times 10^{-3} H$

$C = 24 \times 10^{-6} F$

(a) $X_C = \frac{1}{\omega C} = \underline{\underline{16.57863991 \Omega}}$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{220^2 + (\underbrace{2\pi(400) \cdot 150 \times 10^{-3} - 16.5786\cdots}_{\text{Notice : } X_L > X_C})^2}$

$= \underline{\underline{422.2524774 \Omega}}$

(c) $i_{max} = \frac{\mathcal{E}_m}{Z} = \underline{\underline{0.5210152912 \text{ amp}}}$

(d)
(e)
(f)

$C_{eq} = \frac{1}{\frac{1}{C} + \frac{1}{C}} = \underline{12 \mu F}$

C_{eq} is a half of the original

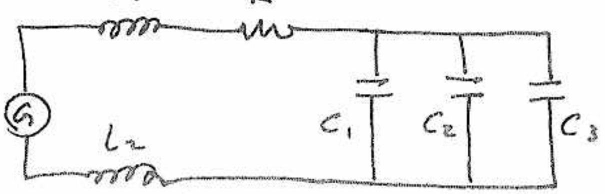
→ $X_C = \frac{1}{\omega C}$ → $\underline{\underline{\text{Doubled}}}$

→ Since $X_L > X_C$ before, $(X_L - X_C)$ is less, then $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is less.

→ $\underline{\underline{Z \text{ should decrease}}}$

→ then $\underline{\underline{i \text{ should increase}}}$

#49



$R = 100 \Omega$

$L_1 = 1.7 \times 10^{-3} H$

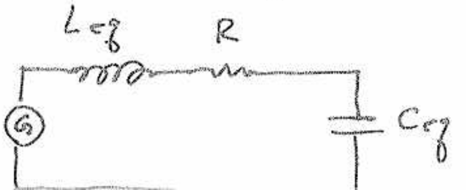
$L_2 = 2.3 \times 10^{-3} H$

$C_1 = 4 \times 10^{-6} F$

$C_2 = 2.5 \times 10^{-6} F$

$C_3 = 3.5 \times 10^{-6} F$

$\Downarrow$



(a) $L_{eq} = L_1 + L_2$ (Series) $= 4 \times 10^{-3} H$

$C_{eq} = C_1 + C_2 + C_3$ (parallel) $= 10 \times 10^{-6} F$

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{4 \times 10^{-3} \cdot 10 \times 10^{-6}}} = 5000 \text{ rad/sec}$$

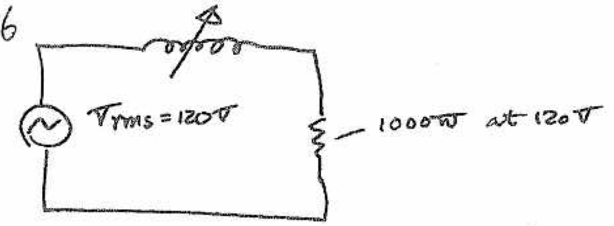
$$\omega = 2\pi\nu \rightarrow \nu = \frac{\omega}{2\pi} = \frac{5000}{2\pi} = \underline{\underline{795.7747155 \text{ Hz}}}$$

(b) Since $\omega_R = \sqrt{\frac{1}{LC}}$ is independent from R, resonant freq. will not change.

(c) If L_1 increases, ω_R decreases $\rightarrow$ $\underline{\underline{\nu_{res.} \text{ also decreases.}}}$

(d) If C_3 is gone, C_{eq} decreases $\rightarrow$ ω_R increases & $\underline{\underline{\nu_{res} \text{ increases}}}$

#56



$$V = IR \rightarrow I = \frac{V}{R}$$
$$P = I \cdot V \longrightarrow P = \frac{V^2}{R} \therefore R = \frac{V^2}{P}$$
$$= \frac{(120)^2}{1000}$$
$$= \underline{\underline{14.4 \Omega}}$$

(a) $$\frac{P_{max}}{P_{min}} = 5 = \frac{i_{max}^2 R}{i_{min}^2 R} = \frac{\left(\frac{V_{rms}}{Z_{min}}\right)^2 \cdot R}{\left(\frac{V_{rms}}{Z_{max}}\right)^2 \cdot R} = \frac{Z_{max}^2}{Z_{min}^2}$$

$$= \frac{R^2 + (\omega L)^2}{R^2}$$

$$\therefore 5R^2 = R^2 + (\omega L)^2$$

$$L^2 = \frac{4R^2}{\omega^2} = \frac{4R^2}{(2\pi\nu)^2} = \frac{4(14.4)^2}{4\pi^2 \cdot 60^2} = \underline{\underline{7.639437268 \times 10^{-2} H}}$$

(b), (c)

modifying $5 = \frac{R^2 + (\omega L)^2}{R^2}$ from (a),

$$5 = \frac{(R_{light} + R)^2}{R_{light}^2}$$

$$5 R_{light}^2 = (R_{light} + R)^2$$

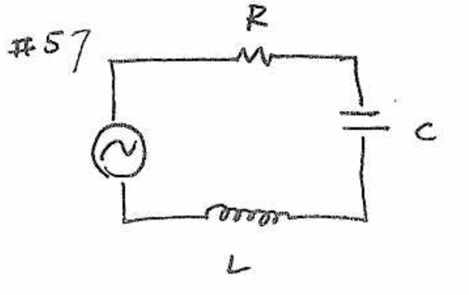
$$\sqrt{5} R_{light} = R_{light} + R$$

$$\therefore R = (\sqrt{5}-1)\,R_{Light}$$

$$= (\sqrt{5}-1)\,(14.4) = \underline{\underline{17.79937888\ \Omega}}$$ yes, it can be done

(d) In (a), L stores energy as magnetic energy (which is retractable)
In (b), R uses energy and it can not be retracted.

#57



$R = 5\Omega$

$L = 60\times10^{-3}\,H$

$f_d = 60\ Hz$

$\mathcal{E}_m = 30\ V$

$i_{rms} = \dfrac{V_{rms}}{Z}$

(a)

$$\overline{P} = (i_{rms})^2 \cdot R$$

We need to minimize Z, $\rightarrow \sqrt{R^2+(X_L-X_C)^2}$ minimized.

$$\rightarrow X_L - X_C = 0 \quad \rightarrow \quad \omega L - \frac{1}{\omega C} = 0$$

$$\therefore \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi\nu)^2 \cdot L} = \underline{\underline{1.172698885\times10^{-4}\,F}}$$

(b) To minimize P, i should be minimized $\rightarrow Z = \infty$. $\rightarrow X_C \rightarrow \infty$

$$\rightarrow \frac{1}{\omega C} = \infty \rightarrow \underline{\underline{C = 0}}$$

(c) $P_{max} = (i^{rms}_{max})^2 \cdot R$ (when Z is minimum)

$$= \left(\frac{V_{max}/\sqrt{2}}{R}\right)^2 \cdot R = \frac{V_{max}}{2R} = \underline{\underline{90\ W}}$$

(f) $P_{min} = 0 \cdot R = \underline{\underline{0\ W}}$

P_{max} happens when $i = i_{max}$.

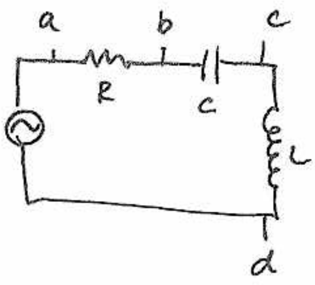
(d) i_{max} happens when Z is min $\rightarrow X_L - X_C = 0 \Rightarrow \tan^{-1}\dfrac{X_L-X_C}{R} = \underline{\underline{0}}$

P_{min} happens when $i = 0$ (No shift!)

(g) $i = 0$ when Z is ∞ $\rightarrow$ $X_L - X_C = -\infty$ (because $C = 0$) $\rightarrow \tan^{-1}\dfrac{X_L-X_C}{R} = \underline{\underline{-\dfrac{\pi}{2}}}$

(e) Power factor : $\cos\phi = 1$ for max (I & V are in sync.)
(h) $\cos\phi = 0$ for min (I & V are Totally off-sync.)

#59



$R = 15\,\Omega$
$C = 4.7\times10^{-6}\,F$
$L = 25\times10^{-3}\,H$
$V_{rms} = 75\,V$
$\nu = 550\,Hz \Rightarrow \omega = 2\pi\nu$

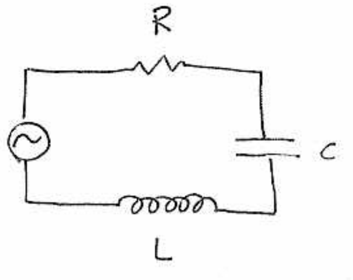
(a)
$$i_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}} = \underline{\underline{2.585763724\ amp}}$$

(b)
$$V_{ab\,rms} = iR = \underline{\underline{38.78645587\,V}}$$
$$V_{bc\,rms} = iX_C = i\left(\frac{1}{\omega C}\right) = \underline{\underline{159.2019764\,V}}$$
$$V_{cd\,rms} = iX_L = i\omega L = \underline{\underline{223.3939488\,V}}$$
$$V_{ad\,rms} = V_{rms} = 75\,V$$

The difference between this (rms) & #42 is either average (rms) or at any given instance. Make sure you can do both.

(c)
$$P_R = i^2R = 100.2926106\,W$$
$$\left.\begin{array}{l} P_C = 0 \\ P_L = 0 \end{array}\right\}$$ they store energy. they do not disapate energy.

#98



$R = 200\,\Omega$
$C = 15\,\mu F$
$L = 230\,mH$
$f_d = 60\,Hz$
$\mathcal{E}_m = 36.0\,V$

(a) $\mathcal{E}_{max} = 36.0\,V$ (Given)

Before we solve for the rest, we need some work

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= 217.3708587\ \Omega$

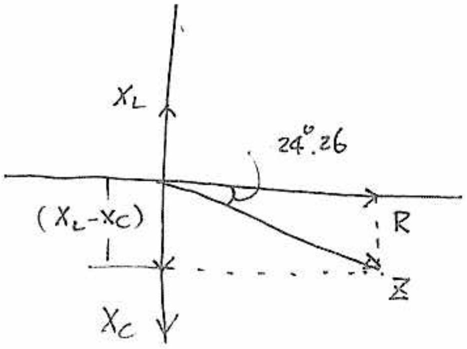
$X_L = \omega L = 2\pi \nu L = 87.70775724\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = 176.8388257\ \Omega$

$\therefore\ i_{max} = \frac{\mathcal{E}_{max}}{Z} = 0.1641056624$ amp

$\phi = \tan^{-1} \frac{X_L - X_C}{R} = -24^\circ.2589527$

Here is the Key point to understand LCR circuit.

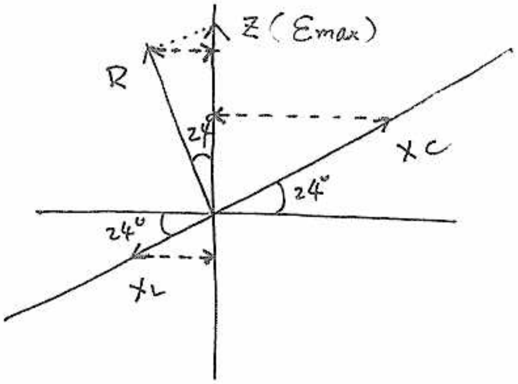


It's very important to draw a phasor diagram as accurately as possible.

This diagram shows that

1. the current (same direction as R) is 24°.26 ahead of V (same direction as Z)
2. y- projections show values of what we are looking for. (In this instant, $i = 0$ and V_{net} = negative, V_L = its max, V_C = its min, and $V_R = 0$.)

The given condition is that $\mathcal{E}$ = max.



So $\mathcal{E}$ direction is on y axis and others follow.

At this moment V across each part is

1. Calculate V_{max} of each.
2. project that value on to the y-axis.

Remember that $\vec{R}$, $\vec{X_C}$ & $\vec{X_L}$ are <u>always</u> rotating. These solutions are true only when $\mathcal{E} = \mathcal{E}_{max}$ (when $\vec{\mathcal{E}}$ is on y axis)

(b) $V_R = i_{max} R \cos 24^\circ ... = 29.92296485\ V$

(c) $V_C = i_{max} X_C \sin 24^\circ ... = 11.9238182\ V$

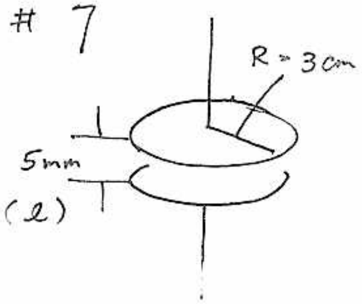
(d) $V_L = i_{max} X_L (-\sin 24^\circ ...) = -5.913671321\ V$ (see the diagram, V_L is down)

(e) +

$35.93311493\ V \sim 36\ V$

ch 32 # 7, 16, 17, 18, 21

7



$\mathcal{E}_m = 150 V$

$\nu = 60 Hz$

$V = 150 \sin(2\pi \nu t)$

Since $V = \int E \cdot ds = E \cdot l$ (E is const.)

$E = \frac{V}{l}$

(a) $\oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

$B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{d(E \cdot A)}{dt}$

$= \mu_0 \varepsilon_0 \pi r^2 \frac{d(E)}{dt}$ ($A = \pi r^2 = const$)

$= \mu_0 \varepsilon_0 \pi r^2 \frac{d(\frac{V}{l})}{dt}$

$= \frac{\mu_0 \varepsilon_0 \pi r^2}{l} \frac{dV}{dt}$

$\therefore B = \frac{\mu_0 \varepsilon_0 \pi r^2}{2\pi r l} \cdot \frac{dV}{dt}$

$\varepsilon_0 = 8.85 \times 10^{-12}$

$\mu_0 = 4\pi \times 10^{-7}$

$r = 0.03 m$

$= \frac{\mu_0 \varepsilon_0}{2l} r \cdot 2\pi\nu \cdot 150 \cancel{\cos(2\pi\nu t)}$ 1 for max B.

$= \underline{\underline{1.886673577 \times 10^{-12} T}}$

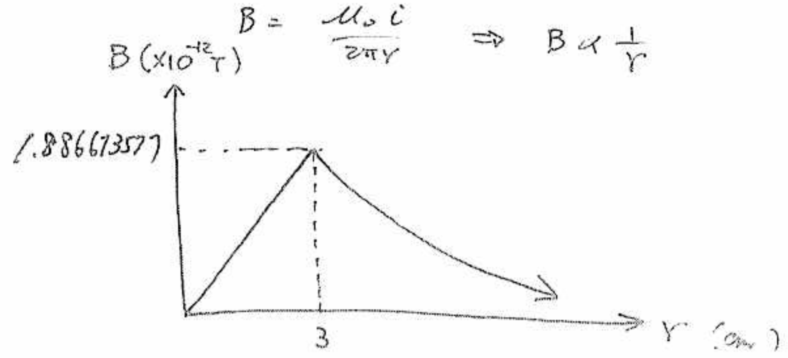
(b) $0 < r < 0.03$

In (a) $B = \frac{\mu_0 \varepsilon}{2l} 2\pi\nu \cdot 150 \cdot r \Rightarrow B \propto r$

$r > 0.03 m$

$B \cdot 2\pi r = \mu_0 i$

$B = \frac{\mu_0 i}{2\pi r} \Rightarrow B \propto \frac{1}{r}$



\# 16

$$E = 4\times10^{5} - 6\times10^{4}\,t$$

$$t = 0\ ,\ E \uparrow$$

$$A = 4\times10^{-2}\,m^{2}$$

(a)
$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$= \varepsilon_0 \frac{A\,dE}{dt} \qquad (A = \text{const.})$$

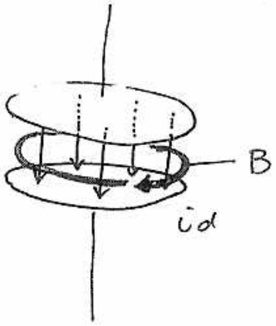
$$= \varepsilon_0 A\,(-6\times10^{4})$$

$$= 8.85\times10^{-12} \cdot 4\times10^{-12} \cdot (-6\times10^{4})$$

$$= \underline{\underline{-2.124\times10^{-8}\,\text{amp}}}$$

(− shows the downward direction)

(b)



Since id is down, B is clockwise seen from above (use the right hand rule)

\# 17

$$\oint B\cdot dS = \mu_0\, i$$

J. current density (A/m²)

(a)
$$B\cdot 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0\, J\cdot A}{2\pi r}$$

$$= \frac{4\pi\times10^{-7}\cdot(20)(\pi\cdot(0.05)^2)}{2\pi\ (0.05)} = \underline{\underline{6.283185307\times10^{-7}\,T}}$$

(b)
$$i_d = J\cdot A = 20\cdot\pi\,(0.05)^2 = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{dE\,A}{dt}$$

$$\therefore \frac{dE}{dt} = \frac{20\cdot\pi\,(0.05)^2}{\varepsilon_0\ \underset{\pi(0.05)^2}{A_R}} = \frac{20}{\varepsilon_0} = \frac{20}{8.85\times10^{-12}} = \underline{\underline{2.259887006\times10^{12}\,\frac{V}{m.s}}}$$

\# 18. (a)
$$i = i_d = i\cdot\frac{\pi\left(\frac{R}{3}\right)^2}{\pi\ R^2} = i\cdot\frac{1}{9} = \underline{\underline{0.i\ \text{Amp}}}$$

(b)
$$B = \frac{\mu_0 i_d}{2\pi r} = \frac{\mu_0\, i\,\frac{\pi r^2}{\pi R^2}}{2\pi r}\quad \frac{\mu_0 i}{2\pi R^2}\cdot r \rightarrow B \propto r$$

(c)
$$B = \frac{\mu_0 i}{2\pi r}\ (\text{outside}) \longrightarrow B \propto \frac{1}{r}$$

Both requires $\frac{12}{3}$ of Bmax or $\frac{1}{4}B_{max}$ ∴ (b) $\frac{1}{4}$ of R $\quad \therefore r = \frac{1.2\,cm}{4} = \underline{\underline{0.3\,cm}}$

(c) 4 of R $\quad r = 4\cdot1.2\,cm = \underline{\underline{4.8\,cm}}$

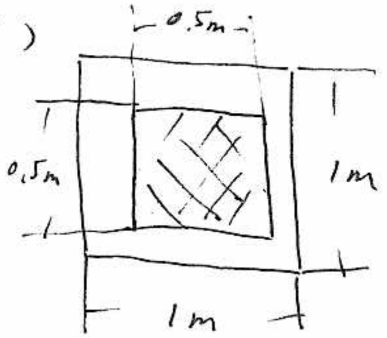
#21 $$\oint B\cdot dS = \mu_0 i + \mu_0 \underbrace{\varepsilon_0 \frac{d\Phi_E}{dt}}_{i_d}$$

(a) 2A $\left(\underset{\text{for outside}}{i} = \underset{\text{for inside}}{i_d} \right)$

(b) $$\varepsilon_0 \frac{d\Phi_E}{dt} = 2A$$

$$\frac{d\Phi_E}{dt} = \frac{2A}{\varepsilon_0} = \frac{2A}{8.85\times10^{-12}} = \underline{\underline{2259887006 \times 10^{11} \frac{V}{m.s}}}$$

(c)



Total Area - $1m \times 1m = 1m^2$

Shaded Area - $0.5m \times 0.5m = 0.25m^2$

$$i_{shaded} = i_d \cdot \frac{\text{Shaded Area}}{\text{Total Area}} = 2 \cdot \frac{0.25m^2}{1m^2}$$

$$= \underline{\underline{0.5A}}$$

(d) $$\int B\cdot dS = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$= \mu_0 \, i_{d(shaded)}$$

$$= 4\pi\times10^{-7} \cdot 0.5A$$

$$= \underline{\underline{0.6283185307 \, \mu T\cdot m}}$$