When you solve any physics problem, follow the steps.

1. Draw an accurate diagram after you read the problem.
2. Pick a point
3. Draw forces, momentum or any appropreate vectors.
4. Write equations about the point (all the equations even if you don't use them all.)
5. Show all of your work.
6. Do not forget units (especially on your final answers!)
7. Do not put an equal sign if they are not equal.

   Ex. Calculate the area of a circle whose diameter is 4.5 cm.

   wrong way: $\frac{4.5\,cm}{2} = (2.25\,cm)^2 \cdot \pi = \underline{\underline{15.9\,cm^2}}$

   correct way: $Area = \pi r^2 = \pi \cdot \left(\frac{4.5\,cm}{2}\right)^2 = \underline{\underline{15.9\,cm^2}}$

8. Carry as many digits as possible (you don't have to write them all) during the calculation. Do not round-off or round-up until the end.

Physics 230

ch. 2. #19, #25, #41, #50, #58, #78, #89, #115

#19. $x = ct^2 - bt^3$

(a) Dimensional Analysis: It is very important. Even though it takes longer, it helps us to see if equations are indeed correct.

$\Rightarrow$ Units are very important!

$$x = c\,t^2 - b\,t^3$$

$$\downarrow \qquad \downarrow\ \downarrow \qquad \downarrow\ \downarrow$$

units $m = (c)(\text{sec})^2 - (b)(\text{sec})^3$

$$\therefore \begin{cases} m = (c)\,\text{sec}^2 & \Rightarrow \underline{\underline{(c) = m/\text{sec}^2}} \\ m = (b)\,\text{sec}^3 & \Rightarrow \underline{\underline{(b) = m/\text{sec}^3}} \end{cases}$$

(b)

$$x = \underset{(m/\text{sec}^2)}{3}t^2 - \underset{(m/\text{sec}^3)}{2}t^3$$

This is a maximization (or minimization) prob.
To solve this type, we take a derivative of the fcn, and set it equals to zero ($\frac{dx}{dt} = 0$). Then check $\frac{dx}{dt}$ (= vel.) before & after the solution. If $\frac{dx}{dt}$ is changing from + to −, then the sol. is max. (If the vel. is changing from positive (going away) to negative (coming back), the peak must be max.)

(c)

$$\frac{dx}{dt} = 2\,(3\ m/sec^2)\,t - 3\,(2\ m/sec^3)\,t^2$$

$$= 6\,t\,(m/sec^2) - 6\,t^2\,(m/sec^3)$$

$$6t - 6t^2 = 0$$

$$6t(1-t) = 0$$

$t = 0$ sec & 1 sec.

t	$\frac{dx}{dt}$
$t < 0$ sec	$-$
0 sec	0
$0 < t < 1$ sec	+
1 sec	0
$t > 1$ sec	$-$

As you can see, at $t = 1$ sec the particle reaches its max.

(d) Distance – (Total movement)

(e) Displacement – (Distance from the origin)

t (sec)	(c) Distance (m)	(d) Displacement (m)
1	1	1
2	5	−4
3	23	−27
4	53	−80
	82 (Total Distance moved from t=0 to 4sec)	

(f) — (m) $V = \frac{dx}{dt} = 6t\,(m/sec^2) - 6t^2\,(m/sec^3)$

$a = \frac{dV}{dt} = 6\,m/sec^2 - 12t\,(m/sec^3)$

t(sec)	V (m/sec)	a (m/sec²)
1	0	-6
2	-12	-18
3	-36	-30
4	-72	-42

#25

Hint: How to start.

$V_0 = 1.5 \times 10^5\, m/sec$

$V_f = 5.7 \times 10^6\, m/sec$

$X = 1.0\ cm = 1.0 \times 10^{-2}\, m$

$a = const$ ——— ①

$V = \int a \cdot dt = at + V_0$ ——— ②

$X = \int V \cdot dt = \int (at + V_0)\,dt = \frac{1}{2}at^2 + V_0 t + \cancel{X_0}^{\,0}$ — ③

For any question, Write down what you know. If it is an eqn, label it.

Eqn ②, Solve for t (Hint: write down what you are going to do → It tells graders that you know what you are doing and it is easier for graders to follow you.)

$V = at + V_0$

$at = V - V_0$

$t = \frac{V - V_0}{a}$ ——— ②'

③ ← ②'

$X = \frac{1}{2}at^2 + V_0 t$

$= \frac{1}{2}a\left(\frac{V - V_0}{a}\right)^2 + V_0\left(\frac{V - V_0}{a}\right)$

$$X = \frac{1}{2} a \frac{(V-V_0)^2}{a^2} + \frac{V_0(V-V_0)}{a}$$

$$= \frac{V^2 - 2VV_0 + V_0^2}{2a} + \frac{2V_0V - 2V_0^2}{2a}$$

$$= \frac{V^2 - V_0^2}{2a}$$

$$\therefore \quad a = \frac{V^2 - V_0^2}{2X}$$

$$= \frac{(5.7\times10^6)^2 - (1.5\times10^5)^2}{2 \cdot 1\times10^{-2}}$$

$$= 1.62 \times 10^{15}\ m/sec^2$$

#41

It is a common sense to use "X" for a horizontal distance and "y" for a vertical distance.

$$a = -g \quad \text{———} \quad ①$$

$$V = \int a.dt = -gt + V_0 \quad \text{———} \quad ②$$

$$y = \int V.dt = -\tfrac{1}{2}gt^2 + V_0t + \cancel{y_0}^{\,0} \text{—} \quad ③$$

at its max. height (50m), V=0.

Eqn. ②

$$0 = -gt + V_0$$

$$t = \frac{V_0}{g} \quad \text{———} \quad ②'$$

③ ← ②'

$$y = -\tfrac{1}{2}gt^2 + V_0t$$

$$50m = -\tfrac{1}{2}g\left(\frac{V_0}{g}\right)^2 + V_0\left(\frac{V_0}{g}\right)$$

$$50 = \frac{1}{2}\frac{V_0^2}{g}$$

$$\therefore V_0 = \sqrt{50 \cdot 2 \cdot g}$$

$$= \underline{\underline{31.32\ {}^{m}/_{sec}}} \quad \text{———} \quad ③'$$

(b)

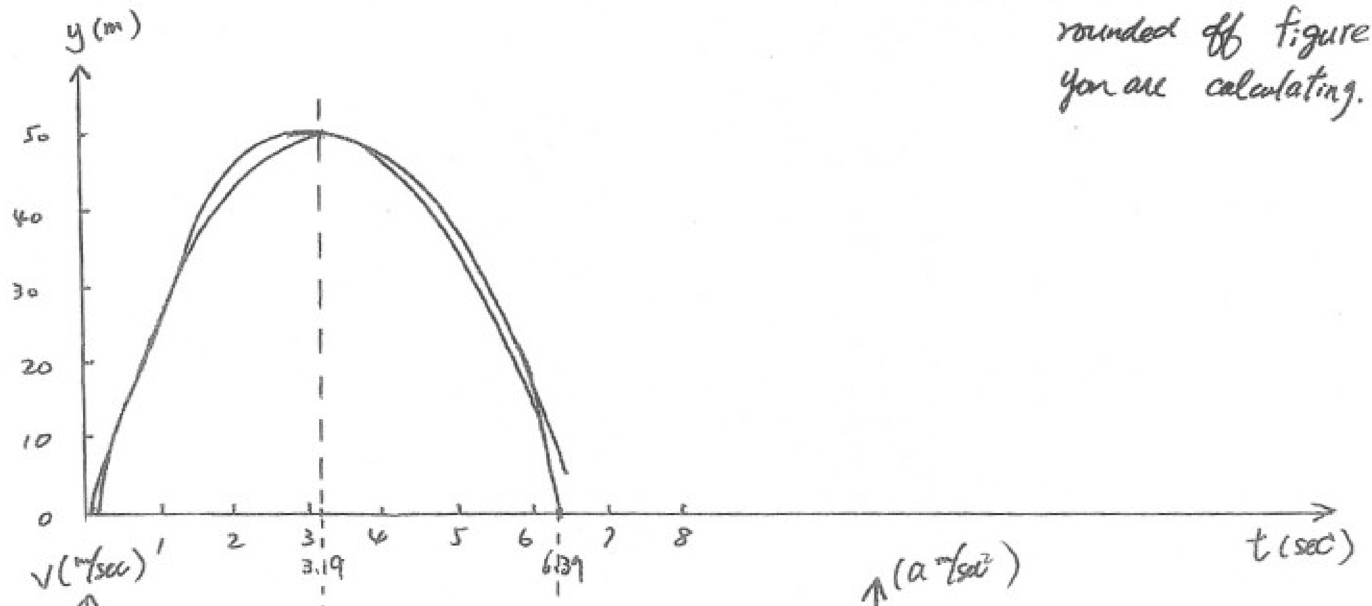
$$②' \longleftarrow ③'$$

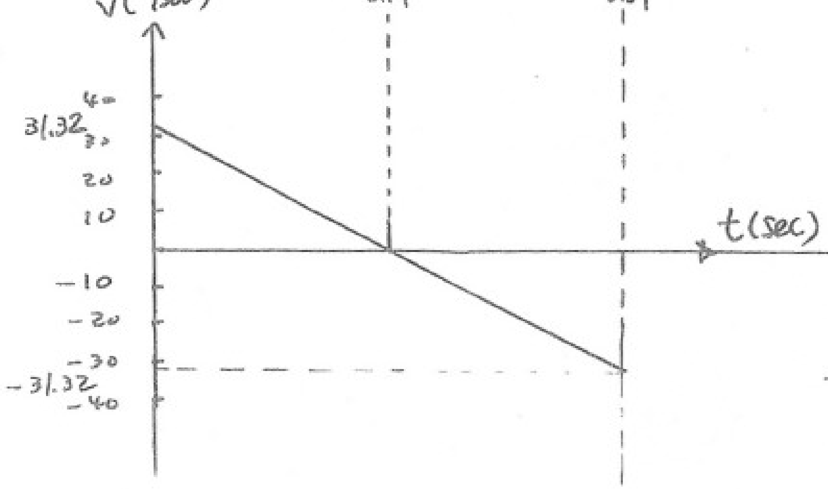
$$t = \frac{\sqrt{50 \cdot 2 \cdot g}}{g} = \sqrt{\frac{50 \cdot 2}{g}} = 3.19\ \text{sec}$$

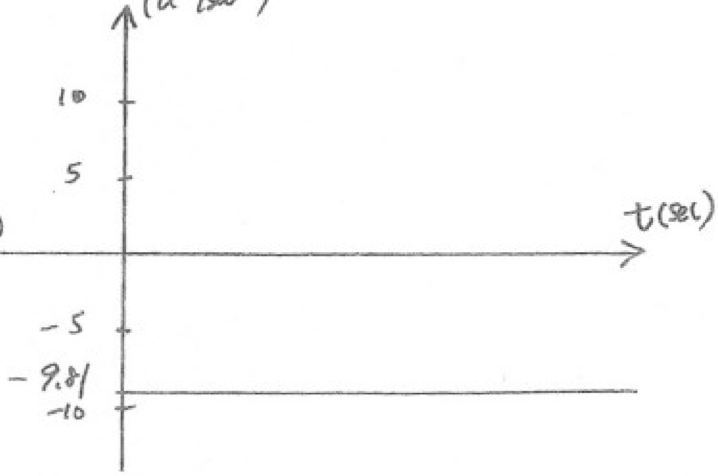
$$\text{time total} = 3.19\ \text{sec} \overset{\text{up}}{\uparrow} + 3.19\ \text{sec} \underset{\text{down}}{\downarrow}$$

$$= \underline{\underline{6.39\ \text{sec} \quad (6.3855..\ \text{sec})}}$$

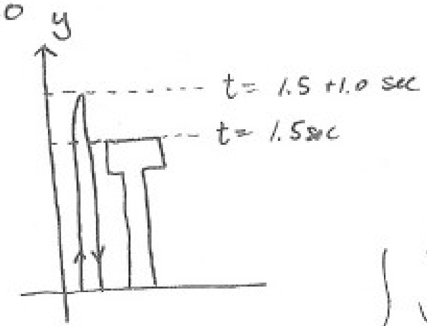
When you write your answer, you round off but do not use rounded off figures when you are calculating.







#50



(max height at 1.5 sec + 1.0 sec = 2.5 sec)

Once again start with basic eqns.

$$\begin{cases} a = -g & \text{—— ①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{—— ②} \\ y = \int V \cdot dt = \int (-gt + V_0)\, dt = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0} & \text{—— ③} \end{cases}$$

Eqn ②, $V = 0$ at $t = 2.5\text{sec}$

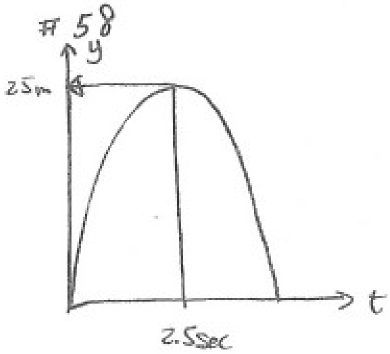
$0 = -gt + V_0$

$\therefore\ V_0 = 2.5g$ —— ②′

③ ← ② @ $t = 1.5\text{sec}$ (to reach the top of the tower)

$y = -\frac{1}{2}g(1.5)^2 + 2.5g(1.5) = \underline{\underline{25.75m\ \ (25.75125\ m)}}$

#58



From the graph, y_{max} is 25m at $t = 2.5$ sec

Even this is on the other planet, physics is still the same!

$$\begin{cases} a = -g & \text{—— ①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{—— ②} \\ y = \int V \cdot dt = \int -gt + V_0 = -\frac{1}{2}gt^2 + V_0 t + \cancel{y_0} & \text{—— ③} \end{cases}$$

(a)

eqn. ② y_{max} ($\equiv V = 0$ at $t = 2.5\text{sec}$)

$0 = -g(2.5) + V_0 \Rightarrow V_0 = 2.5g$ —— ②′

③ ← ②′ ($y = 25m$)

$25 = -\frac{1}{2}g(2.5)^2 + (2.5g)(2.5)$

$= \frac{1}{2}g(2.5)^2$

$\therefore\ g = \frac{2 \cdot 25}{(2.5)^2} = \underline{\underline{8.0\ m/sec^2}}$ —— ③′

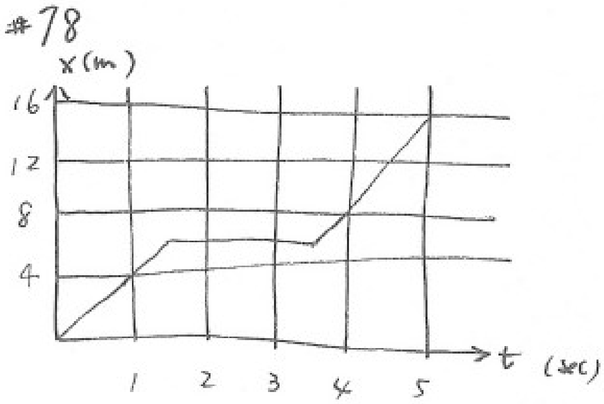
(b)

②′ ← ③′

$V_0 = 2.5g$

$= 2.5\text{sec}(8.0\ m/sec^2) = \underline{\underline{20.0\ m/sec}}$

#78



(a) $x(t = 0.5\,\text{sec}) = 2\,m$

$x(t = 4.5\,\text{sec}) = 12\,m$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{12\,m - 2\,m}{4.5\,\text{sec} - 0.5\,\text{sec}} = \underline{\underline{2.5\ m/\text{sec}}}$$

(b) instantaneous vel $= \frac{dx}{dt}$ = slope of $x(t)$ fcn (as shown on the top)

Notice that the slope is const between $t = 4$ sec & 5 sec (because the line is straight)

$$\therefore \frac{dx}{dt} = \frac{\Delta x}{\Delta t} = \frac{x(t = 5\,\text{sec}) - x(4\,\text{sec})}{5\,\text{sec} - 4\,\text{sec}}$$

$$= \frac{16\,m - 8\,m}{1\,\text{sec}} = \underline{\underline{8\ m/\text{sec}}}$$

(c) $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(t = 4.5\,\text{sec}) - v(t = 0.5\,\text{sec})}{4.5\,\text{sec} - 0.5\,\text{sec}}$$

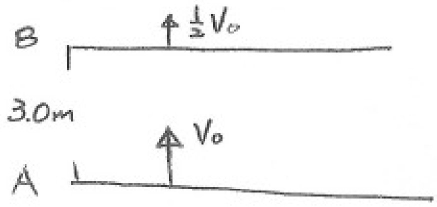
$$= \frac{8\ m/\text{sec} - 4\ m/\text{sec}}{4\,\text{sec}}$$

$v(t = 0.5\,\text{sec})$ is calculated the same way used at $t = 4.5$ sec in (b)

$$= \underline{\underline{1\ m/\text{sec}^2}}$$

(d) Since v is const $a = \frac{dv}{dt} = \underline{\underline{0\ m/\text{sec}^2}}$

\# 89.



let V_0 be the speed at A
Also, let A be the origin.

$$\begin{cases} a = -g & \text{——— ①} \\ V = \int a.dt = -gt + V_0 & \text{——— ②} \\ y = \int v.dt = \int(-gt+V_0)dt = -\frac{1}{2}gt^2 + V_0t + \cancel{y_0}^{\,0} & \text{— ③} \end{cases}$$

(a) at $y = 3.0\,m$, $V = \frac{1}{2}V_0$

Eqn. ②, Solve for t

$$\frac{1}{2}V_0 = -gt + V_0$$

$$t = \frac{V_0}{2g} \quad \text{——— ②'}$$

③ ← ②'

$$3 = -\frac{1}{2}g\left(\frac{V_0}{2g}\right)^2 + V_0\left(\frac{V_0}{2g}\right)$$

$$= -\frac{1}{2}\frac{V_0^2}{4g} + \frac{V_0^2}{2g}$$

$$= -\frac{V_0^2}{8g} + \frac{V_0^2}{2g} = \frac{3}{8}\frac{V_0^2}{g}$$

$$\therefore V_0 = \sqrt{3 \cdot \frac{8g}{3}} = \sqrt{8g} = \underline{\underline{8.859\ m/sec}}$$

(b) at the max. pt., $V = 0$.

Eqn ② Solve for t

$$0 = -gt + V_0$$

$$t = \frac{V_0}{g} \quad \text{——— ②'}$$

③ ← ②'

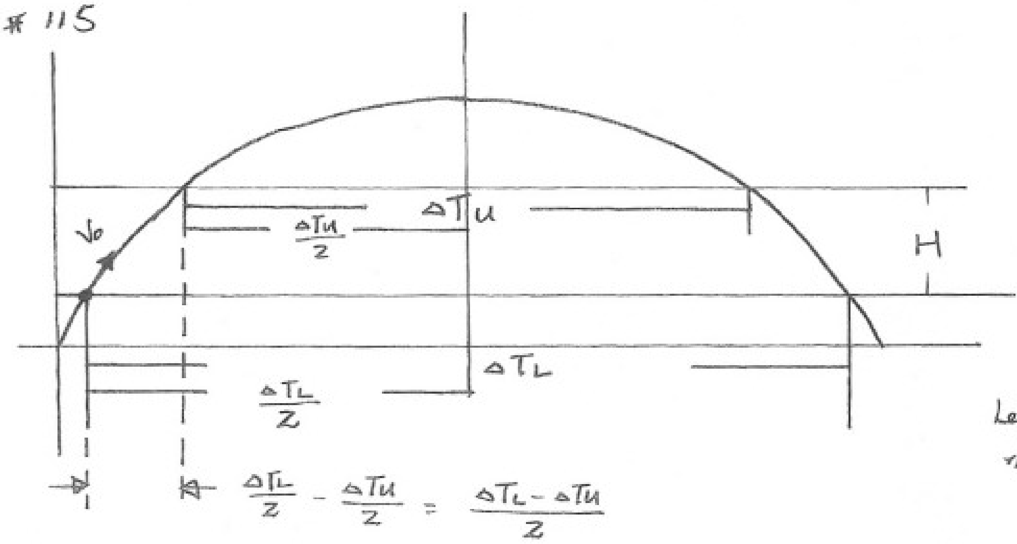
$$y = -\frac{1}{2}gt^2 + V_0t$$

$$= -\frac{1}{2}g\left(\frac{V_0}{g}\right)^2 + V_0\left(\frac{V_0}{g}\right)$$

$$= \frac{1}{2}\frac{V_0^2}{g} = \frac{1}{2}\frac{(8.859)^2}{g} = 4\,m$$

$\therefore$ It is ($4m - 3m = \underline{1m}$) higher than B

115



Because there is only one constant acc. (of g), the left half & right half are symmetric to each other.
$\Rightarrow \frac{1}{2}$ total time to reach the max.

Let V_0 be the speed at the Lower level.

$$\begin{cases} a = -g & \text{①} \\ V = \int a \cdot dt = \int -g \cdot dt = -gt + V_0 & \text{②} \\ y = \int v \cdot dt = \int (-gt + V_0)\, dt = -\frac{1}{2} g t^2 + V_0 t + \cancel{y_0}^{\,0} & \text{③} \end{cases}$$

Eqn ② $\quad V = 0$ at $t = \frac{\Delta T_L}{2}$

$$0 = -g\left(\frac{\Delta T_L}{2}\right) + V_0$$

$$\therefore \quad V_0 = \frac{\Delta T_L\, g}{2} \quad \text{③}'$$

③ $\leftarrow$ ②$'$ & $\quad y = H$ at $t = \frac{\Delta T_L - \Delta T_U}{2}$

$$H = -\frac{1}{2} g \left(\frac{\Delta T_L - \Delta T_U}{2}\right)^2 + \frac{\Delta T_L\, g}{2}\left(\frac{\Delta T_L - \Delta T_U}{2}\right)$$

$$= -\frac{g}{8}(\Delta T_L - \Delta T_U)^2 + \frac{\Delta T_L\, g}{4}(\Delta T_L - \Delta T_U)$$

$$= \left(-\frac{1}{8}\left(\Delta T_L^2 - 2\Delta T_L \Delta T_U + \Delta T_U^2\right) + \frac{\Delta T_L^2 - \Delta T_L \Delta T_U}{4}\right) g$$

$$= \frac{1}{8}\left(-\Delta T_L^2 + 2\Delta T_L \cdot \Delta T_U - \Delta T_U^2 + 2\Delta T_L^2 - 2\Delta T_L \Delta T_U\right) g$$

$$= \frac{1}{8}\left(\Delta T_L^2 - \Delta T_U^2\right) g$$

$$\therefore \quad g = \frac{8H}{\Delta T_L^2 - \Delta T_U^2}$$

ch 3 # 31, # 37, # 49

#31 Given:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\Theta = a_x b_x + a_y b_y + a_z b_z \quad \text{— (1)}$$

$$\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$$

$$\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$$

$$\therefore |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2}$$

$$= \sqrt{27} \quad \text{— (2)}$$

$$|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$$= \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2}$$

$$= \sqrt{14} \quad \text{— (3)}$$

$$(1) \leftarrow (2), \& (3)$$

$$|\vec{a}||\vec{b}| \cos\Theta = a_x b_x + a_y b_y + a_z b_z$$

$$\sqrt{27}\sqrt{14} \cos\Theta = (3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)$$

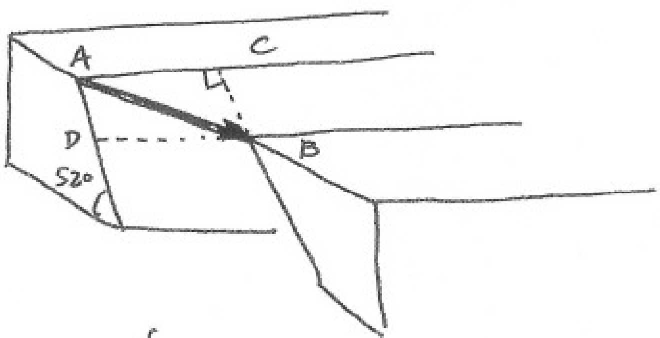
$$\therefore \cos\Theta = \frac{18}{\sqrt{27}\sqrt{14}}$$

$$\therefore \Theta = \cos^{-1}\frac{18}{\sqrt{27}\sqrt{14}}$$

$$= 22.2076543$$

$$\sim \underline{\underline{22.^\circ 2}}$$

37



pt. B is still touching the fault plane.

(a) If it slid from A to B

$\overline{AC} = 22m$

$\overline{AD} = 17m$

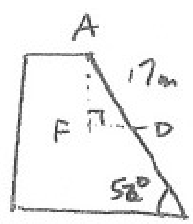
Hence $\vec{AB} = \vec{AC} + \vec{AD}$

and $|\vec{AB}| = \sqrt{(\vec{AC})^2 + (\vec{AD})^2}$

$= \sqrt{(22m)^2 + (17m)^2}$

$= \underline{\underline{27.8\,m}}$

(b)



If $|\vec{AD}| = 17m$ and $\measuredangle = 52°$

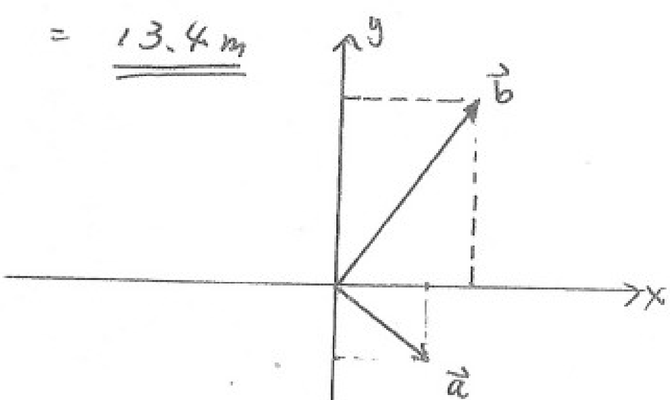
then $\overline{AF} = 17m \cdot \sin 52°$

$= \underline{\underline{13.4\,m}}$

49.

$\vec{a} = (4.0m)\hat{i} - (3.0m)\hat{j}$

$\vec{b} = (6.0m)\hat{i} + (8.0m)\hat{j}$



It is always a good idea to draw a diagram

(a) $|\vec{a}| = \sqrt{a_i^2 + a_j^2}$

$= \sqrt{(4.0m)^2 + (-3.0m)^2}$

$= \underline{\underline{5\,m}}$

(b) $\alpha = \tan^{-1}\frac{a_j}{a_i} = \tan^{-1}\frac{-3}{4} = \underline{\underline{-36.87°}}$

(c) $|\vec{b}| = \sqrt{(6m)^2 + (8m)^2}$

$= \underline{\underline{10\ m}}$

(d) $\beta = \tan^{-1}\frac{8}{6} = \underline{\underline{53.^\circ 13}}$

(e) $\vec{a} + \vec{b} = (4\hat{i} - 3\hat{j}) + (6\hat{i} + 8\hat{j})$

$= (4+6)\hat{i} + (-3+8)\hat{j}$

$= 10_m\, i + 5_m \hat{j}$

$|\vec{a}+\vec{b}| = \sqrt{(10_m)^2 + (5_m)^2}$

$= \underline{\underline{11.18\ m}}$

(f) $\gamma = \tan^{-1}\frac{5}{10} = \underline{\underline{26.^\circ 57}}$

(g) $\vec{b} - \vec{a} = (6\hat{i} + 8\hat{j}) - (4\hat{i} - 3\hat{j})$

$= (6-4)\hat{i} + (8+3)\hat{j}$

$= \underline{\underline{2_m\hat{i} + 11_m\hat{j}}}$

(h) $\delta = \tan^{-1}\frac{11}{2} = \underline{\underline{79.^\circ 70}}$

(i) $\vec{a} - \vec{b} = (4\hat{i} - 3\hat{j}) - (6\hat{i} + 8\hat{j})$

$= (4-6)\hat{i} + (-3-8)\hat{j}$

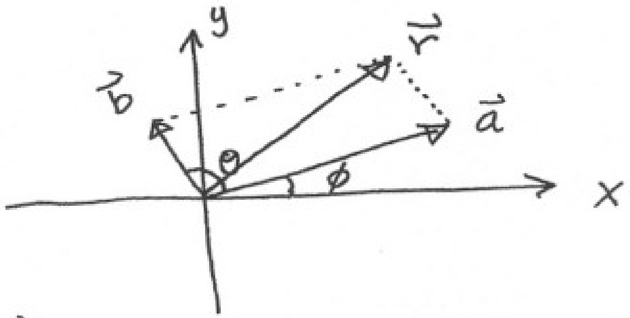
$= \underline{\underline{-2\,m\hat{i} - 11\,m\hat{j}}}$

(j) $\varepsilon = \tan^{-1}\frac{-11}{-2} = 259.^\circ 7$ (be careful. Since x & y are both negative the vector is in the 3rd Quad.)

(k) As you can see they are opposite to each other.

→ 180° apart.

Extra



$$\vec{a} = a_x + a_y = |a|\cos\phi + |a|\sin\phi$$
$$\vec{b} = b_x + b_y = |b|\cos(\phi+\theta) + |b|\sin(\phi+\theta)$$

$$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x) + (a_y + b_y)$$

and

$$|\vec{r}| = \sqrt{(a_x+b_x)^2 + (a_y+b_y)^2}$$

$$= \sqrt{\left(|a|\cos\phi + |b|\cos(\phi+\theta)\right)^2 + \left(|a|\sin\phi + |b|\sin(\phi+\theta)\right)^2}$$

$$= \sqrt{a^2\cos^2\phi + b^2\cos^2(\phi+\theta) + 2|a||b|\cos\phi\cos(\phi+\theta) + a^2\sin^2\phi + b^2\sin^2(\phi+\theta) + 2|a||b|\sin\phi\sin(\phi+\theta)}$$

$$= \sqrt{a^2\underbrace{(\cos^2\phi + \sin^2\phi)}_{1} + b^2\underbrace{(\cos^2(\phi+\theta) + \sin^2(\phi+\theta))}_{1} + 2|a||b|(\cos\phi\cos(\phi+\theta) + \sin\phi\sin(\phi+\theta))}$$

trig. Identity.

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

so

$$\cos(a-b) = \cos a\cos(-b) - \sin a\sin(-b)$$
$$= \cos a\cos b + \sin a\sin b.$$

$a = \phi$

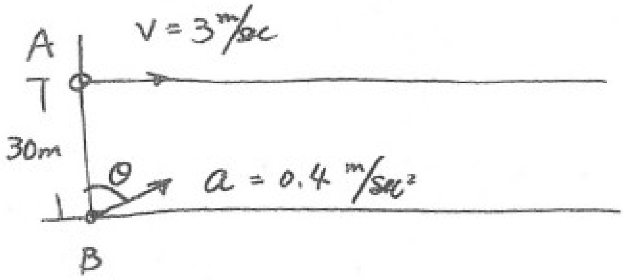
$b = \phi + \theta.$

$$= \sqrt{a^2 + b^2 + 2ab\cos(\phi - (\phi+\theta))}$$

$$= \sqrt{a^2 + b^2 + 2ab\cos(-\theta)} = \underline{\underline{\sqrt{a^2 + b^2 + 2ab\cos\theta}}}$$

Ch. 4 #16, #19, #26, #85, #96

#16



A

x comp

$a_{Ax} = 0$ —— ①

$V_{Ax} = V = 3\,m/sec$ —— ②

$X_A = Vt = 3t$ —— ③

y comp

$a_{Ay} = 0$ —— ④

$V_{Ay} = 0$ —— ⑤

$y_A = 30\,m$ —— ⑥

B

x comp

$a_{Bx} = a\sin\theta$ —— ⑦

$V_{Bx} = at\sin\theta$ —— ⑧

$X_B = \frac{1}{2}at^2\sin\theta$ —— ⑨

y comp

$a_{By} = a\cos\theta$ —— ⑩

$V_{By} = at\cos\theta$ —— ⑪

$y_B = \frac{1}{2}at^2\cos\theta$ —— ⑫

At collision

$X_A = X_B$ (③=⑨) &

$y_A = y_B$ (⑥=⑫)

$3t = \frac{1}{2}at^2\sin\theta$

$30 = \frac{1}{2}at^2\cos\theta$ — ⑥′

$3 = \frac{1}{2}at\sin\theta$

$\therefore\ t = \frac{6}{a\sin\theta}$ — ③′

⑥′ ← ③′

$30 = \frac{1}{2}a\left(\frac{6}{a\sin\theta}\right)^2\cos\theta$

$60 = \frac{6^2}{a\sin^2\theta}\cos\theta$

$5a \sin^2\theta = 3\cos\theta$ (since $a = 0.4\ m/sec^2$) $5a = 2\ m/sec^2$

$2(1-\cos^2\theta) - 3\cos\theta = 0$

$2\cos^2\theta + 3\cos\theta - 2 = 0.$

$$\cos\theta = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2\cdot 2}$$

$$= \frac{2}{4} \text{ or } \cancel{-2} \text{ not possible}$$

$\therefore\ \theta = \cos^{-1}\frac{1}{2} = \underline{\underline{60°}}$

#19

x comp

$a_x = 0$ ——— ①

$V_x = \int a_x\,dt = V_{ox}$ ——— ②

$X = \int V_x\,dt = V_{ox}\,t$ ——— ③

y comp

$a_y = -g$ ——— ④

$V_y = \int a_y \cdot dt = -gt + V_{oy}$ ——— ⑤

$y = \int V_y \cdot dt = -\frac{1}{2}gt^2 + V_{oy} + y_o$ ——— ⑥

at $y = 9.1\,m$, $V_x = 7.6\ m/sec$ & $V_y = 6.1\ m/sec$

There are two different ways to put the origin. Since it asks the total horizontal distance, I will use the ground as the origin (then, $y_o = 0\,m$)

Eqn. ⑤, $V_y = 6.1\ m/sec$. Solve for t

$6.1 = -gt + V_{oy}$

$t = \dfrac{V_{oy} - 6.1}{g}$ ——— ⑤'

⑥ ⟵ ⑤' ($y = 9.1\,m$ at that moment)

$$9.1 = -\frac{1}{2}gt^2 + V_{oy}\,t$$

$$= -\frac{1}{2}g\left(\frac{V_{oy}-6.1}{g}\right)^2 + V_{oy}\left(\frac{V_{oy}-6.1}{g}\right)$$

$$= \frac{V_{oy}^2 - 6.1^2}{2g}$$

$\therefore\ V_{oy} = \sqrt{(9.1)(2)(g) + 6.1^2}$

$= 14.69 \quad (14.6884989)\ m/sec$ ——— ⑥'

(a) At the top, $V = 0\ m/sec$

Eqn. ⑤, solve for t

$0 = -gt + V_{oy}$

$t = \frac{V_{oy}}{g}$ ——— ⑤'

⑥ ← ⑤'

$y = -\frac{1}{2}gt^2 + V_{oy}t$

$= -\frac{1}{2}g\left(\frac{V_{oy}}{g}\right)^2 + V_{oy}\left(\frac{V_{oy}}{g}\right)$

$= -\frac{1}{2}g\frac{V_{oy}^2}{g^2} + \frac{V_{oy}^2}{g}$

$= \frac{1}{2}\frac{V_{oy}^2}{g} = \underline{\underline{10.9965\ m}}$

(b) Eqn ⑥, $y = 0$, solve for t

$0 = -\frac{1}{2}gt^2 + V_{oy}t$

$t\left(-\frac{1}{2}gt + V_{oy}\right) = 0$

$-\frac{1}{2}gt + V_{oy} = 0$

$t = \frac{2V_{oy}}{g} = 2.994597126\ sec$ ——— ⑥'

③ ← ⑥'

$x = V_{ox}t$

$= 7.6 \times 2.99\ldots$

$= \underline{\underline{22.76\ m}}$

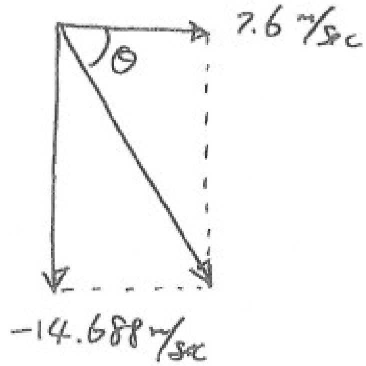
(c) ② & ⑤ ← ⑥'

$V_x = 7.6\ m/sec$ (No acc. toward x direction)

$V_y = -gt + V_{oy} = -14.6884989\ m/sec$

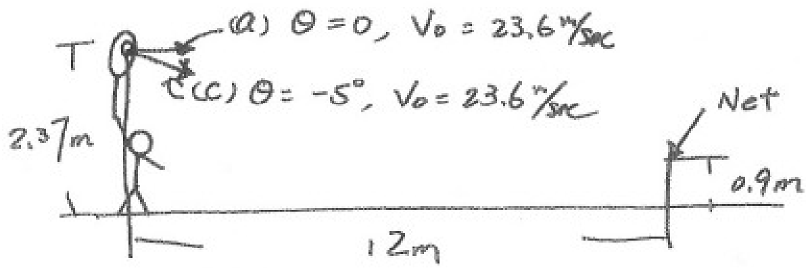
$$|V| = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(7.6)^2 + (-14.688)^2} = \underline{\underline{16.538 \text{ m/sec}}}$$



$$\theta = \tan^{-1} \frac{V_y}{V_x} = \underline{\underline{-62.°64}}$$

#26.



(a) $\theta = 0°$

$\underline{x \text{ comp}}$

$a_x = 0$ ——— ①

$V_x = V_0 = 23.6 \text{ m/sec}$ ——— ②

$X = V_0 t$ ——— ③

$\underline{y \text{ comp}}$

$a_y = -g$ ——— ④

$V_y = -gt + \cancel{V_{0y}}^{\,0}$ ——— ⑤

$y = -\frac{1}{2} g t^2 + \cancel{y_0}^{\,2.37}$ ——— ⑥

Eqn. ③, $X = 12 m$, solve for t

$12 = V_0 t$

$t = \frac{12}{V_0}$ ——— ③'

⑥ ⟵ ③'

$y = -\frac{1}{2} g t^2 + 2.37$

$= -\frac{1}{2} g \left(\frac{12}{V_0}\right)^2 + 2.37$

$= -1.26817 + 2.37$

$= 1.1018299356 m$ ... Yes, it does clear the net

(b) $(1.1018 - 0.9) m = 0.2018 m$ (~ 20 cm above the net → Bad serve!)

(c) $\theta = -5°$

x comp

$a_x = 0$ ——— ①

$V_x = V_0 \cos(-5°) = V_0 \cos 5°$ ——— ②

$X = V_0 t \cos 5°$ ——— ③

y comp

$a_y = -g$ ——— ④

$V_y = -gt + V_0 \sin(-5°)$

$= -gt - V_0 \sin 5°$ ——— ⑤

$y = -\frac{1}{2}gt^2 - V_0 t \sin 5° + 2.37$ ——— ⑥

Eqn ③, $X = 12\,m$, solve for t

$12 = V_0 t \cos 5°$

$t = \frac{12}{V_0 \cos 5°}$ ——— ③'

⑥ ⟵ ③'

$$y = -\frac{1}{2} g \left(\frac{12}{V_0 \cos 5°}\right)^2 - \cancel{V_0}\left(\frac{12}{\cancel{V_0}\cos 5°}\right)\sin 5° + 2.37$$

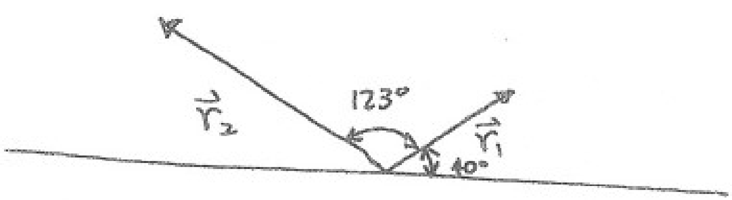
$$= -\frac{1}{2} g \frac{12^2}{(23.6 \cos 5°)^2} - 12 \tan 5° + 2.37$$

$$= -1.27787697 - 1.0499 + 2.37 = -2.32774094\,m + 2.37 = 0.0423\,m$$

⇒ No it does not clear the net.

$0.9 - 0.0423 = 0.8577\,m$ (~86 cm below the net → Fault!)

#85.



$\vec{r}_1 = 360\,m,\ 40°$

$\vec{r}_2 = 790\,m,\ (40° + 123°)$

$\vec{r}_1 = |\vec{r}_1| \cos 40°\,\hat{i} + |\vec{r}_1| \sin 40°\,\hat{j}$

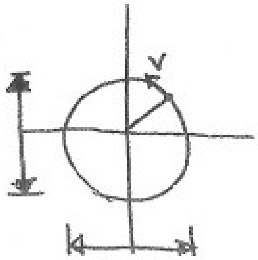
$\vec{r}_2 = |\vec{r}_2| \cos(40° + 123°)\,\hat{i} + |\vec{r}_2| \sin(40° + 123°)\,\hat{j}$

$$\text{Displacement} = \sqrt{(r_{2x} - r_{1x})^2 + (r_{2y} - r_{1y})^2}$$

$$= \sqrt{\left((790)\cos 163° - (360)\cos 40°\right)^2 + \left((790)\sin 163° - (360)\sin 40°\right)^2}$$

$$= \underline{\underline{1031.26\,m}}$$

#96.



Basic circular eqn.

a circular motion is a combination of x motion — $A\cos(\omega t+\phi)$ & y motion — $A\sin(\omega t+\phi)$, where A is an amplitude, ω is an angular speed, and ϕ is a phase shift. If you are not sure of these, you might want to brush up "Trig."

In this case, $r = 3m \Rightarrow A = 3m$

Period = 20 sec $\Rightarrow$ 2π rad takes 20 sec $\Rightarrow \omega = \frac{2\pi \text{ rad}}{20\text{ sec}} = \frac{\pi}{10}$ rad/sec

$x_{origin} = 0$

y_{origin} = 3m below the basic eqn.

$\Rightarrow y = 3 + A\sin(\omega t + \phi)$

at $t = 0$ sec, the particle is at $(0,0)$ $\Rightarrow$ $\frac{\pi}{2}$ rad behind the basic eqn. $\Rightarrow \phi = -\frac{\pi}{2}$

So.

$$\begin{cases} x = 3\cos\left(\frac{\pi}{10}t - \frac{\pi}{2}\right) \\ y = 3 + 3\sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right) \end{cases}$$

(a) $x\,(t = 5\text{sec}) = 3\cos\left(\frac{\pi}{10}(5) - \frac{\pi}{2}\right) = 3m$

$y\,(t = 5\text{sec}) = 3 + 3\sin\left(\frac{\pi}{10}(5) - \frac{\pi}{2}\right) = 3m$ $(3, 3)$

$$\underline{\vec{r} = 3_m\hat{i} + 3_m\hat{j}}$$

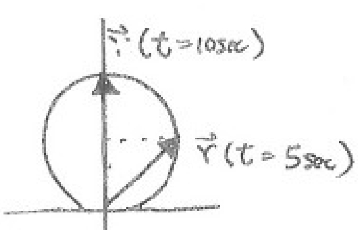
(b) $x(t = 7.5\text{sec}) = 2.121\text{ m}$

$y(t = 7.5\text{sec}) = 5.121\text{ m}$ $\Big\}$ $\underline{\vec{r} = 2.121\text{m}\,\hat{i} + 5.121\text{ m}\,\hat{j}}$

(c) $x\,(t = 10\text{sec}) = 0\text{ m}$

$y\,(t = 10\text{sec}) = 6\text{ m}$ $\Big\}$ $\underline{\vec{r} = 0\text{m}\,\hat{i} + 6\text{ m}\,\hat{j}}$

(d)

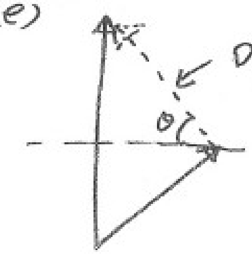


Displacement $= \sqrt{\Delta x^2 + \Delta y^2}$

$= \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$

$= \sqrt{(0-3m)^2 + (6m-3m)^2} = \sqrt{9+9}\,m = \underline{\underline{3\sqrt{2}\,m}}$

(e)



As you can see, since $\Delta x = -3m$ & $\Delta y = 3m$,

$\underline{\underline{\theta = 45^\circ}}$ speed $= \dfrac{\Delta D}{\Delta t} = \dfrac{3\sqrt{2}\,m}{5} = 0.6\sqrt{2}\,m/sec$

$= \underline{\underline{0.8485\,m/sec}}$

you can do this by calculating V_x & V_y separately.

(f) & (g)

$V_x = \dfrac{dx}{dt} = -3\dfrac{\pi}{10}\sin\left(\dfrac{\pi}{10}t - \dfrac{\pi}{2}\right) \qquad V_y = \dfrac{dy}{dt} = 3\dfrac{\pi}{10}\cos\left(\dfrac{\pi}{10}t - \dfrac{\pi}{2}\right)$

$\left.\begin{array}{l} V_x(t=5sec) = 0\,m/sec \\ V_y(t=5sec) = \dfrac{3\pi}{10}\,m/sec \end{array}\right\} \quad \underline{\underline{V = 0\,m/sec\,\hat{i} + \dfrac{3\pi}{10}\,m/sec\,\hat{j}}}$

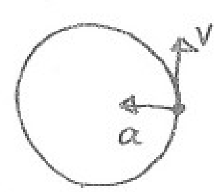
$\left.\begin{array}{l} V_x(t=10sec) = -\dfrac{3\pi}{10}\,m/sec \\ V_y(t=10sec) = 0\,m/sec \end{array}\right\} \quad \underline{\underline{V = -\dfrac{3\pi}{10}\,m/sec\,\hat{i} + 0\hat{j}}}$

$\left\{\begin{array}{l} a_x = \dfrac{dV_x}{dt} = -3\dfrac{\pi^2}{100}\cos\left(\dfrac{\pi}{10}t - \dfrac{\pi}{2}\right) \\ a_y = \dfrac{dV_y}{dt} = -3\dfrac{\pi^2}{100}\sin\left(\dfrac{\pi}{10}t - \dfrac{\pi}{2}\right) \end{array}\right.$

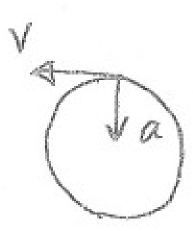
$\left.\begin{array}{l} a_x(t=5sec) = -\dfrac{3\pi^2}{100}\,m/sec^2 \\ a_y(t=5sec) = 0\,m/sec^2 \end{array}\right\} \quad \underline{\underline{a = -\dfrac{3\pi^2}{100}\,m/sec^2\,\hat{i} + 0\,m/sec^2\,\hat{j}}}$

$\left.\begin{array}{l} a_x(t=10sec) = 0\,m/sec^2 \\ a_y(t=10sec) = -\dfrac{3\pi^2}{100}\,m/sec^2 \end{array}\right\} \quad \underline{\underline{a = 0\,m/sec^2\,\hat{i} - \dfrac{3\pi^2}{100}\,m/sec^2\,\hat{j}}}$

These results are expected because at $t = 5sec$, the particle is at (3,3). V is always tangent. So V at that point has to have only positive y and its acc. should have only negative x.



At $t = 10$ sec, the particle is at (0,6). V should be negative x direction only and a should be negative y only.

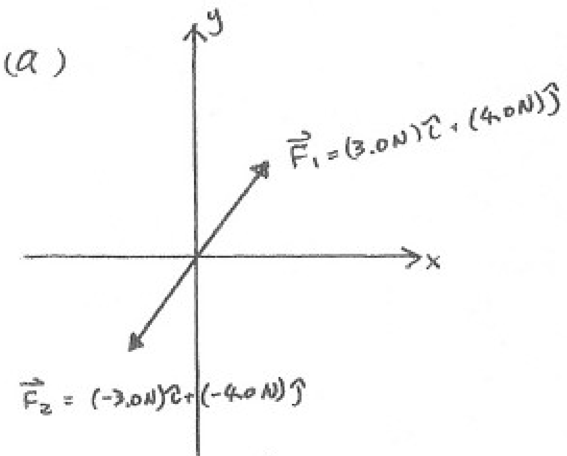


Ch 5. #3, #8, #42, #51, #72, #82, #83

#3. Warning: You still are a beginner. You can't be lazy yet. Try to draw an accurate diagram & write equations step by step.

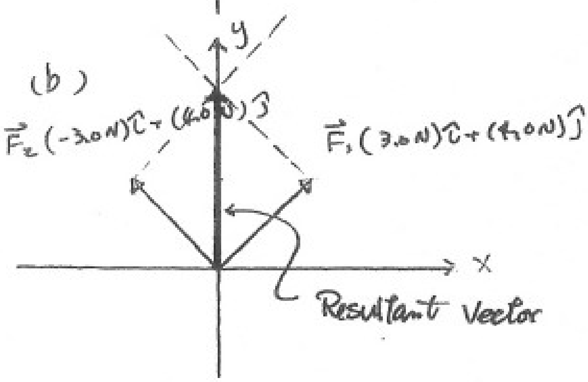
$m = 2.0\ \text{kg}$

(a)



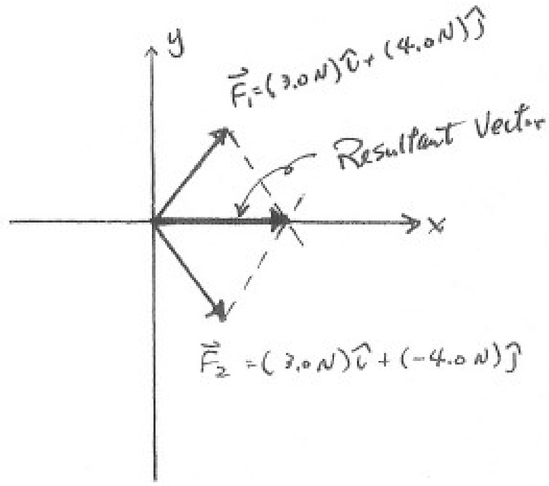
$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2$$
$$= (3.0N)\hat{i} + (4.0N)\hat{j} + (-3.0N)\hat{i} + (-4.0N)\hat{j}$$
$$= (3.0-3.0)N\hat{i} + (4.0-4.0)N\hat{j}$$
$$= 0N\hat{i} + 0N\hat{j} \quad \text{(Equilibrium)}$$
$$\therefore \vec{a} = \frac{\vec{F}}{m} = \frac{0N}{2.0kg}\hat{i} + \frac{0N}{2.0kg}\hat{j} = \underline{\underline{0\ m/sec^2\ \hat{i} + 0\ m/sec^2\ \hat{j}}}$$

(b)



$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2$$
$$= (3.0N)\hat{i} + (4.0N)\hat{j} + (-3.0N)\hat{i} + (4.0N)\hat{j}$$
$$= (3.0-3.0)N\hat{i} + (4.0+4.0)N\hat{j}$$
$$= 0N\hat{i} + 8.0N\hat{j}$$
$$\therefore \vec{a} = \frac{\vec{F}}{m} = \frac{0N}{2.0kg}\hat{i} + \frac{8.0N}{2.0kg}\hat{j} = \underline{\underline{0\ m/sec^2\ \hat{i} + 4\ m/sec^2\ \hat{j}}}$$

(c)



$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2$$
$$= (3.0N)\hat{i} + (4.0N)\hat{j} + (3.0N)\hat{i} + (-4.0N)\hat{j}$$
$$= (3.0+3.0)N\hat{i} + (4.0-4.0)N\hat{j}$$
$$= 6.0N\hat{i} + 0N\hat{j}$$
$$\therefore \vec{a} = \frac{\vec{F}}{m} = \frac{6.0N}{2.0kg}\hat{i} + \frac{0N}{2.0kg}\hat{j} = \underline{\underline{3\ m/sec^2\ \hat{i} + 0\ m/sec^2\ \hat{j}}}$$

\# 8

From the given diagram, we can calculate a_x, and hence we can calculate F_x as well.

Notice that the line passes (0 sec, -4 m/sec) & (2 sec, 2 m/sec)

$$\therefore \quad a_x = \frac{\Delta V_x}{\Delta t} = \frac{2\,\text{m/sec} - (-4\,\text{m/sec})}{2\,\text{sec} - 0\,\text{sec}} = \underline{\underline{3\,\text{m/sec}^2}}$$

$$\therefore \Sigma F_x = m a_x = 4.0\,\text{kg} \cdot 3.0\,\text{m/sec}^2 = 12\,\text{N}$$

Since the two forces ($\vec{F}_1$ & $\vec{F}_2$) are acting on the 4.0 kg and the net force is 12N & $\vec{F}_1 = 7.0\,\text{N}\,\hat{i} + 0.0\,\text{N}\,\hat{j}$,

Method 1

$$\Sigma F_x = 12\,\text{N} = F_{1x} + F_{2x}$$

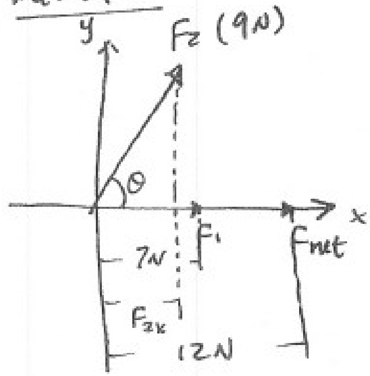
$$= 7.0\,\text{N} + |\vec{F}_2| \cos\theta$$

$$\therefore \quad 7.0\,\text{N} + 9.0\,\text{N} \cos\theta = 12\,\text{N}$$

$$\therefore \cos\theta = \frac{5}{9}$$

$$\therefore \theta = \cos^{-1}\frac{5}{9} = 56.°251014 \sim \underline{\underline{56.°3}}$$

Method 2



$$\vec{F}_1 = 7.0\,\text{N}\,\hat{i} + 0\,\text{N}\,\hat{j}$$

$$\vec{F}_2 = F_{2x}\,\hat{i} + F_{2y}\,\hat{j} \quad \& \quad |\vec{F}_2| = 9\,\text{N}$$

$$\Sigma F_x = 12\,\text{N} = F_{1x} + F_{2x}$$

$$= 7.0\,\text{N} + F_{2x} \qquad \therefore \underline{\underline{F_{2x} = 5.0\,\text{N}}}$$

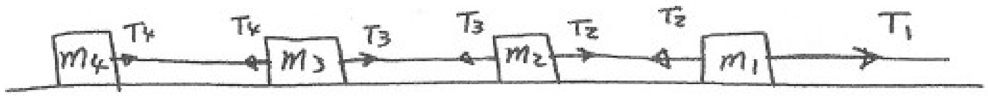
Also

$$|\vec{F}_2| = 9.0\,\text{N} = \sqrt{F_{2x}^2 + F_{2y}^2}$$

$$= \sqrt{(5\,\text{N})^2 + (F_{2y})^2}$$

$$\therefore \quad F_{2y} = \sqrt{(9.0\,\text{N})^2 - (5.0\,\text{N})^2} = \sqrt{56}\,\text{N}$$

$$\therefore \quad \theta = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{\sqrt{56}}{5} = 56.°2510114 \sim \underline{\underline{56.°3}}$$



$T_1 = 222\,N$
$T_3 = 111\,N$

Method 1 (pick a point)

At m_1 $\quad T_1 - T_2 = m_1 a$ (Tension difference between T_1 & T_2 causes m_1 to acc.)
At m_2 $\quad T_2 - T_3 = m_2 a$ (" T_2 & T_3 " m_2 ")
At m_3 $\quad T_3 - T_4 = m_3 a$ (" T_3 & T_4 " m_3 ")
At m_4 $\quad T_4 = m_4 a$ (Tension 4 causes m_4 to acc.)

$$\begin{cases} 222 - T_2 = 20a & \text{—— ①} \\ T_2 - 111 = 15a & \text{—— ②} \\ 111 - T_4 = m_3 a & \text{—— ③} \\ T_4 = 12a & \text{—— ④} \end{cases}$$

Eqn. ①. solve for T_2

$T_2 = -20a + 222$ —— ①′

② ⟵ ①′ solve for a

$(222 - 20a) - 111 = 15a \Rightarrow a = \frac{111}{35}$ —— ②′

③ ⟵ ②′ & ④

$111 - 12a = m_3 \frac{111}{35}$

$111 - 12 \cdot \frac{111}{35} = m_3 \frac{111}{35} \Rightarrow m_3 = \dfrac{\left(111 - 12 \cdot \frac{111}{35}\right)}{\frac{111}{35}} = \underline{\underline{23\,kg}}$

Method 2.

T_1, 222N of force caused 4 penguin to accelerate.

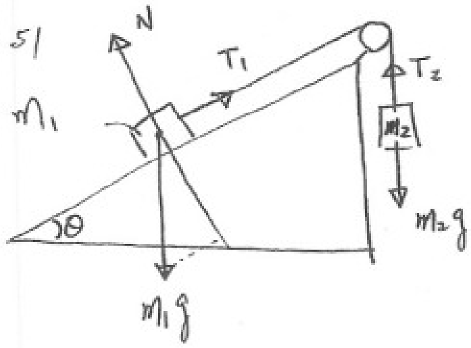
$222N = (20kg + 15kg + m_3 + 12kg)\,a$ —— ①

T_3, 111 N of force caused m_3 & m_4 to accelerate

$111\,N = (m_3 + 12kg)\,a$ —— ②

After playing with these eqns., you will get the same result.

51



$m_1 = 3.70\,kg$
$m_2 = 2.30\,kg$
$\theta = 30°$

Initial test :

$m_1 g \sin\theta \overset{?}{\lesseqgtr} m_2 g$

$1.85\,kg < 2.3\,kg$

the initial test indicates that m_1 will slide up

(a) m_1

x (along the slope)

$-m_1 g \sin\theta + T_1 = m_1 a$ — ①

y (extra for this problem)

$-m_1 g \cos\theta + N = 0$

m_2

x

None

y

$-m_2 g + T_2 = -m_2 a$ — ②

Since $T_1 = T_2$, from eqn. ②

$T_2 = T_1 = -m_2 a + m_2 g$ — ②'

① ← ②'

$-m_1 g \sin\theta + (-m_2 a + m_2 g) = m_1 a$

$-m_1 g \sin\theta + m_2 g = m_1 a + m_2 a$

$\therefore a = \dfrac{(-m_1 \sin\theta + m_2) g}{(m_1 + m_2)} = \dfrac{(-3.7 \sin 30° + 2.3)\, 9.81}{(3.7 + 2.3)}$

$= 0.73575\ m/sec^2$ (upward)

(b) The initial test is very important & helpfull & saves your time. Make sure you do the test → upward

(c) eqn ①

$T_1 = m_1 a + m_1 g \sin\theta = m_1 (a + g \sin\theta)$

$= 3.70\,(0.73575 + 9.81 \sin 30°)$

$= 20.870775\,N$

\#57 This is not our 230 Level problem, but I just wanted to stress the importance of indicating "units".

(a) $7682\,L \times \frac{1.77\,kg}{1\,L} = \underline{\underline{13597.14\,kg}}$

(b) $(22300\,kg - 13597.14\,kg) \times \frac{1\,L}{1.77\,kg} = \underline{\underline{4916.870056\,L}}$

(c) $7682\,L \times \frac{1.77\,lb}{1\,L} \times \frac{1\,kg}{2.205\,lb} = \underline{\underline{6166.503401\,kg}}$ (1 kg = 2.205 lb from Appendix A-6)

(d) $(22300\,kg - 6166.503401\,kg) \times \frac{2.205\,lb}{1\,kg} \cdot \frac{1\,L}{1.77\,lb} = \underline{\underline{20098.50867\,L}}$

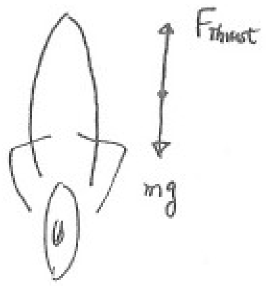
(e) $\dfrac{7682\,L + 4916.870056\,L \text{ (asked to add)}}{22300\,kg \times \frac{2.205\,lb}{1\,kg} \cdot \frac{1\,L}{1.77\,lb} \text{ (Total needed)}} \times 100$

$= \underline{\underline{45.35\,\%\text{ of the Max. fuel capacity}}}$

(they were Lucky this time, but usually things are not this Lucky.)

\#72 Even though the things are not on the earth, physics is physics. start w/ diagrams, pick a pt. or pts., draw forces, and write an eqn or eqns.

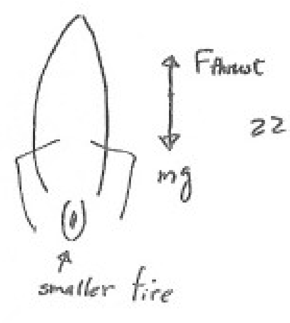
Case 1



$3260\,N - mg = m\cancel{a}^{\,0}$ — ①

(Because it descends at const. speed $\Rightarrow a = 0$)

Case 2



$2200\,N - mg = ma$ — ② ($a = -0.39\,m/sec^2$)

(a) Eqn ①

$3260\,N - mg = 0$

$\therefore \underline{\underline{mg = 3260\,N}} = \text{weight}$ — ①'

(b) ② ← ①'

$2200\,N - mg = -0.39\,m$

$\therefore \underline{\underline{m = 2717.948718\,kg}}$ — ②'

(c) ①' ← ②'

$g = \frac{3260\,N}{m} = \underline{\underline{1.199433962\,m/sec^2}}$

(a)

$a = \text{const.}$ —— ①

$V = \int a \cdot dt = at + \cancel{V_0}^{0}$ —— ② $\quad m_s = 1.20 \times 10^6 \, kg$

$X = \int v \cdot dt = \frac{1}{2} a t^2 + \cancel{X_0}^{0}$ —— ③

Eqn. ② $\quad V = 0.1C = 3 \times 10^8 \, m/sec \times 0.1 = 3 \times 10^7 \, m/sec$

$t = 3 \text{ days} = 3 \text{ days} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ sec}}{1 \text{ min}}$

$\therefore a = \frac{v}{t} = \frac{3 \times 10^7 \, m/sec}{3 \times 24 \times 60 \times 60 \, sec} = \underline{\underline{1.1574 \times 10^2 \, m/sec^2}}$ —— ②′

(b) $1.1574 \times 10^2 \, m/sec^2 \times \frac{1 \, g}{9.81 \, m/sec^2} = \underline{\underline{11.80 \, g\text{'s}}}$ (Wait! Normal people die under this acc.)

(c) $1 \text{ light sec} = 3 \times 10^8 \, m$

$\Rightarrow 5 \text{ light months} = 3.888 \times 10^{15} \, m$

Dist. the ship travels in 1st 3days (during the acc.)

③ ⟵ ②′

$X = \frac{1}{2} (1.1574 \times 10^2)(3 \cdot 24 \cdot 60 \cdot 60 \, sec)^2$

$= 3.888 \times 10^{12} \, m$

The rest ($3.888 \times 10^{15} m - 3.888 \times 10^{12} m$) is traveled at $V = 0.1C$.

$t = \frac{D}{t} = \frac{(3.888 \times 10^{15} m - 3.888 \times 10^{12} m)}{3 \times 10^7 \, m/sec} = 1.2947 \times 10^8 \, sec$

$= 1.4985 \times 10^3 \text{ days}$

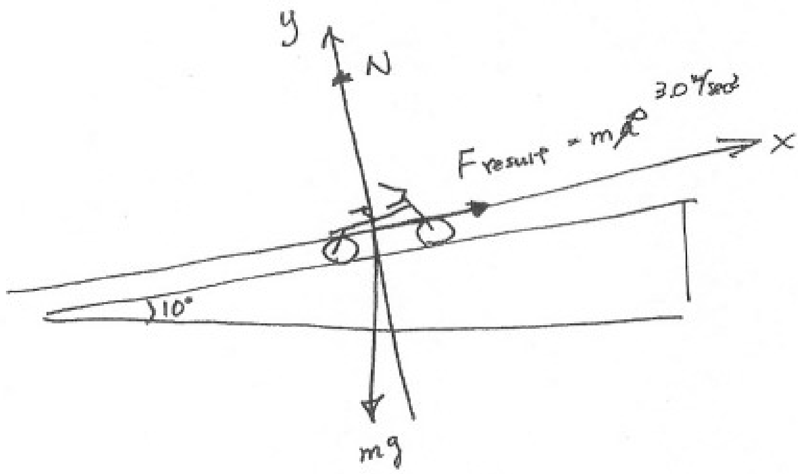
$= 4.105479452 \text{ yrs}$

$= 4 \text{ yrs } 38.5 \text{ days}.$

Total time $= 3 \text{ days} + 4 \text{ yrs}. 38.5 \text{ days}$

$= \underline{\underline{4 \text{ yrs}. 41.5 \text{ days}}}$

#83



X comp

$-mg\sin\theta + F_{bike,x} = ma$ ——— ①

y comp

$-mg\cos\theta + N = 0$ ——— ②

(a) Resultant Force (along the inclined plane - $\Sigma F_y = 0$)

$ma = 60.0\ kg \cdot 3.0\ m/sec^2 = \underline{180\ N}$

(b) Equ ①, solve for $F_{bike,x}$

x: $F_{bike,x} = ma + mg\sin\theta$

$= 60\ kg \cdot 3.0\ m/sec^2 + 60\ kg \cdot 9.81\ m/sec^2 \cdot \sin 10°$

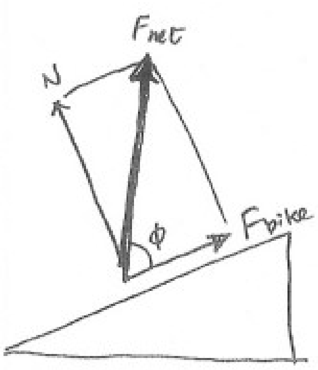
$= \underline{282.2093174\ N}$

y. Equ. ②. solve for N.

$N = mg\cos\theta$

$= 60\ kg \cdot 9.81\ m/sec^2 \cos 10°$

$= \underline{579.6578434\ N}$



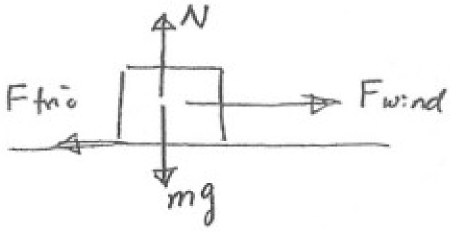
$|F_{net}| = \sqrt{(F_{bike,x})^2 + N^2}$

$= \underline{644.7\ N}$ (644.7056028 N)

$\phi = \tan^{-1}\dfrac{579.6\cdots}{282.2\cdots} = \underline{64.04° \text{ above the plane}}$

ch. 6 # 4, 7, 8, 20, 21, 26, 35, 49, 52, 53

#4



$m = 20\,kg$
$\mu_k = 0.8$

x

$F_{wind} - F_{fric} = m\cancel{a}^{0}$ — ①

$F_{wind} - N\mu_k = 0$

y

$N - mg = 0$

$\therefore N = mg$ — ②

① ← ②

$F_{wind} - mg\,\mu_k = 0$

$F_{wind} = mg\,\mu_k = 20\,kg \cdot 9.81\,m/sec^2 \cdot 0.8$

$= \underline{\underline{156.96\,N}}$

#35

(a) Eqn. 6-14 $D = \frac{1}{2} C\rho A V^2$ where $C = 0.8$; $\rho = 1.21\,kg/m^3$ and $A = 0.040\,m^2$ in this case.

So this drag force (156.96 N from #4) has to come from the force of wind — moving air particles.

Solve Eqn. 6-14 for V.

$$D = \frac{1}{2} C\rho A V^2$$

$$\therefore V = \sqrt{\frac{2D}{C\rho A}} = \sqrt{\frac{2 \cdot 156.96\,N}{0.8 \cdot 1.21 \cdot 0.04}} = 90.041\,m/sec$$

$$= \underline{324.15\,km/hr}$$

(b) $324.15\,km/hr \times 2 = \underline{\underline{648.30\,km/hr}}$ $(402.67\,mi/hr)$

It is impossible w/ this given condition.

the error occured in #4 & #32 is μ_k of the clay.

Clay is very slick when wet. It should've been 0.15 for μ_k.

If this was the case, the ans. for (a) is 99.25 km/hr ... little less than

[illegible]

\# 53

(a)

F needed for a rock   156.96 N (from # 4)

F needed for 100 rocks   156.96 N × 100 = 15696 N

F needed for the ice sheet (Eqn. is the same as #4)

$$\overset{\mu_k}{F = 0.1}\,(\overset{m}{400_m \times 500_m \times 40\times 10^{-3}_m \times 917\ \text{kg/m}^3})\cdot \overset{g}{9.81}$$

$$= 791661.6\ N$$

Total Force Needed   15696 N + 791661.6 N = $\underline{735357.6\ N}$

Since $D = 4\, C_{ice}\, \rho\, A_{ice}\, V^2$

$$V = \sqrt{\frac{D}{4\, C_{ice}\, \rho\, A_{ice}}}$$

$$= \sqrt{\frac{735357.6}{4 \cdot 2\times 10^{-3} \cdot 1.21 \cdot 400\times 500}}$$

$$= 19.48931684\ \text{m/sec} = \underline{\underline{70.16\ \text{km/hr}}}$$

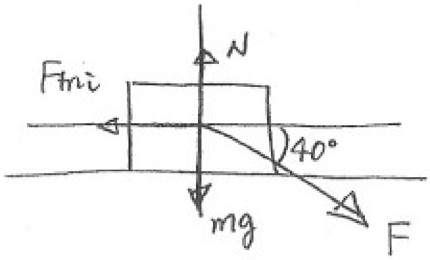
(b)

$$70.16\ \text{km/hr} \times 2 = \underline{\underline{140.32}}\ \text{km/hr}$$

(c)

This is possible and reasonable.

\#7



$m = 3.5\,kg$

$F = 15\,N$

$\theta = 40°$

$\mu_k = 0.25$

Don't forget the y-comp of F.

x

$F\cos 40° - F_{fric} = ma$.

$\therefore F\cos 40° - N\mu_k = ma$ —①

y

$N - mg - \underline{F\sin 40°} = 0$

$\therefore N = mg + F\sin 40°$ —②

(a) Since $F_{fric} = N\mu_k \leftarrow$ ②

$= (mg + F\sin 40°)\,0.25$

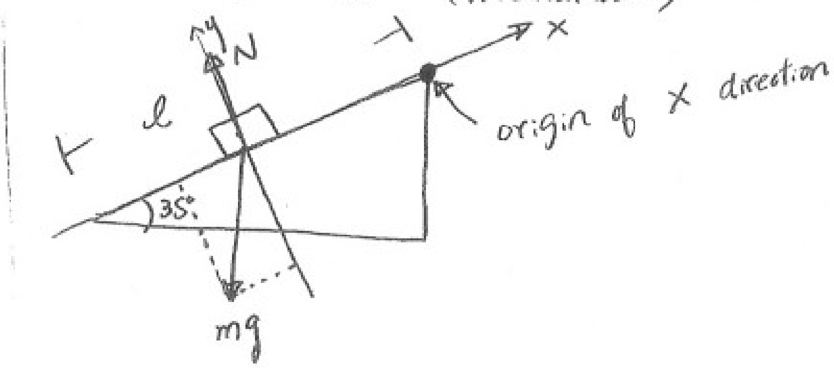
$= \underline{\underline{10.994\,N}}$ ——— ②′

(b) ① ← ②′

$F\cos 40° - 10.994\,N = ma$

$\therefore a = \dfrac{F\cos 40° - 10.994\,N}{m} = \underline{\underline{0.1418\ m/sec^2}}$

\# 8 Case I (frictionless)



x

$F_x = ma_x = -mg\sin\theta$

$\therefore a_x = -g\sin\theta$ ——— ①

$V_x = -gt\sin\theta$ ——— ②

$X = -\frac{1}{2}gt^2\sin\theta$ ——— ③

y

$F_y = -mg\cos\theta + N$

$\therefore a_y = -g\cos\theta + \frac{N}{m} = 0$ —④

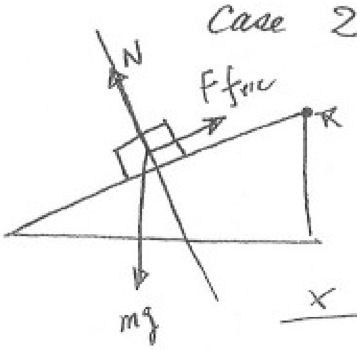
Eqn. ③ $X = -l$ (start at the top. Hence the dist. is negative)

Solve for t

$$-\ell = -\frac{1}{2} g t^2 \sin\theta$$

$$t = \sqrt{\frac{2\ell}{g\sin\theta}} \quad \text{—— ③'}$$

Case 2 (w/ friction)



once again, this is the origin

x	y
$F_x = -mg\sin\theta + F_{fric}$	$F_y = -mg\cos\theta + N = 0$
$ma_x = -mg\sin\theta + N\mu_k$ —— ⑤	$\therefore N = mg\cos\theta$ —— ⑥

⑤ ← ⑥

$$ma_x = -mg\sin\theta + \mu_k mg\cos\theta$$

$$a_x = -g\sin\theta + \mu_k g\cos\theta = g(-\sin\theta + \mu_k\cos\theta) \quad \text{—— ⑤'}$$

$$V_x = \int a_x \cdot dt' = gt'(-\sin\theta + \mu_k\cos\theta) \quad \text{—— ⑦}$$

$$x = \int v_x \cdot dt = \frac{1}{2} g t'^2 (-\sin\theta + \mu_k\cos\theta) \quad \text{—— ⑧}$$

Eqn. ⑧. $x = -\ell$

$$-\ell = \frac{1}{2} g t'^2 (-\sin\theta + \mu_k\cos\theta) \quad \text{—— ⑧'}$$

since $t' = 2t$ (frictionless takes a half time)

⑧' ← ③'

$$-\ell = \frac{1}{2} g\left(2\sqrt{\frac{2\ell}{g\sin\theta}}\right)^2 (-\sin\theta + \mu_k\cos\theta)$$

$$-\cancel{\ell} = \frac{1}{\cancel{2}} \cancel{g}\left(4\,\frac{\cancel{2}\cancel{\ell}}{\cancel{g}\sin\theta}\right)(-\sin\theta + \mu_k\cos\theta)$$

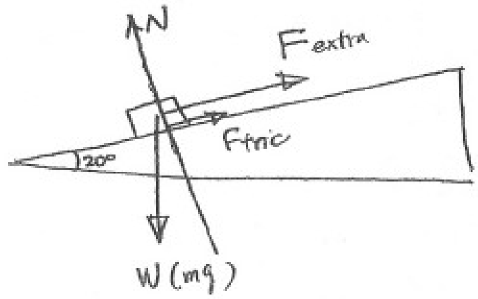
$$-1 = \frac{4}{\sin\theta}(-\sin\theta + \mu_k\cos\theta)$$

$$-\frac{\sin\theta}{4} + \sin\theta = \mu_k\cos\theta$$

$$\mu_k = \frac{\frac{3}{4}\sin\theta}{\cos\theta} = \underline{\underline{0.525}}$$

(0.52515563)

#20



$W = mg = 80\,N$

$\mu_s = 0.25$

$\mu_k = 0.15$

6-5

(a)

X

$F_{extra} + F_{tric} - mg\sin\theta = 0$

$F_{extra} + N\mu_s - mg\sin\theta = 0$ —— ①

y

$N - mg\cos\theta = 0$

$\therefore N = mg\cos\theta$ —— ②

① ⟵ ②

$F_{extra} + (mg\cos\theta)\mu_s - mg\sin\theta = 0$

$\therefore F_{extra} = mg\sin\theta - mg\,\mu_s\cos\theta$

$= mg(\sin\theta - \mu_s\cos\theta) = \underline{8.57\,N}\ (8.567759\!\cdot\!05)$

(b) In order for the sled to move upward, the force has to break the friction (downward). So the eqns. become

F_x

$-mg\sin\theta - F_{tric} + F = 0$

$\therefore -mg\sin\theta - N\mu_s + F = 0$ —— ①

F_y

$N - mg\cos\theta = 0$

$N = mg\cos\theta$ —— ②

① ⟵ ②

$-mg\sin\theta - \mu_s mg\cos\theta + F = 0$

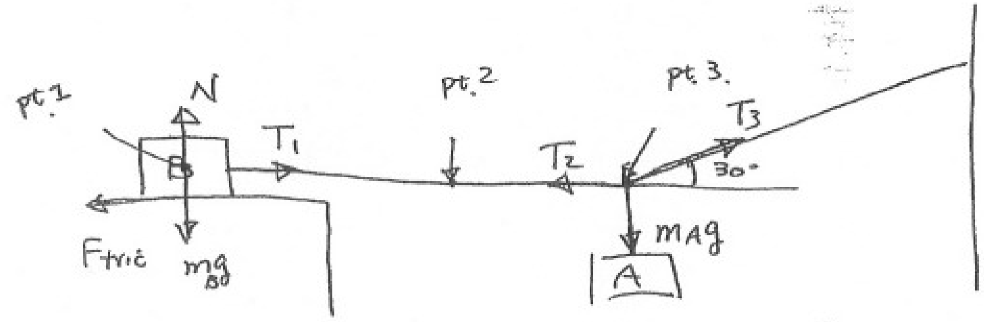
$F = mg\sin\theta + \mu_s mg\cos\theta$

$= mg(\sin\theta + \mu_s\cos\theta) = \underline{46.16\,N}\ (46.15546388\,N)$

(c) Once the sled starts moving, the coeff. is μ_k. Eqns. are all the same in (b) except for μ_k

$\therefore F = mg(\sin\theta + \mu_k\cos\theta) = \underline{38.64\,N}\ (38.63792292)$

#21



pt 1

x

$-F_{tric} + T_1 = 0$

$\therefore -N\mu_s + T_1 = 0$ — ①

y

$N - m_B g = 0$

$\therefore N = m_B g$ — ②

① ← ②

$-m_B g \mu_s + T_1 = 0$

$\therefore T_1 = m_B g \mu_s$ — ①'

pt 2

$T_1 - T_2 = 0$

$\therefore T_1 = T_2$

pt. 3

x

$-T_2 + T_3 \cos 30° = 0$ — ③

y

$T_3 \sin 30° - m_A g = 0$ — ④

③ ← ①'

$-m_B g \mu_s + T_3 \cos 30° = 0$

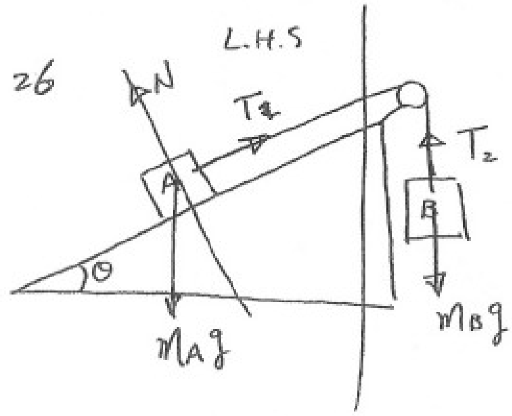
$\therefore T_3 = \dfrac{m_B g \mu_s}{\cos 30°}$ — ③'

④ ← ③'

$\dfrac{m_B g \mu_s}{\cos 30°} - m_A g = 0$

$\therefore m_A = \dfrac{m_B g \mu_s}{g \cos 30°} = \underline{\underline{102.624 N}}$

\# 26



$m_A = 10\,kg$

$\theta = 30°$

$\mu_k = 0.2$

L. H. S.

x comp

$-m_A g \sin\theta + T_1 + F_{fric} = m\cancel{a}^{0}$

$\therefore -m_A g \sin\theta + T_1 + N\mu_k = 0$ —— ①

y comp

$-m_A g \cos\theta + N = 0$

$\therefore N = m_A g \cos\theta$ —— ②

$T_1 = T_2$ —— ③

R. H. S

x comp

None

y

$-m_B g + T_2 = 0$

$T_2 = m_B g$ —— ④

① ← ②, ③ & ④

$-m_A g \sin\theta + m_B g + m_A g \cos\theta \cdot \mu_k = 0$

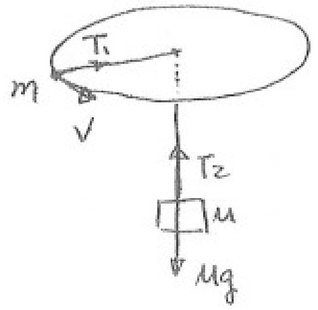
solve for m_B

$m_B g = m_A g \sin\theta - m_A g \mu_k \cos\theta$

$= m_A g (\sin\theta - \mu_k \cos\theta)$

$\therefore m_B = 10\,(\sin\theta - \mu_k\cos\theta) = \underline{\underline{3.27\,kg}}$ (3.267949192…)

\# 49



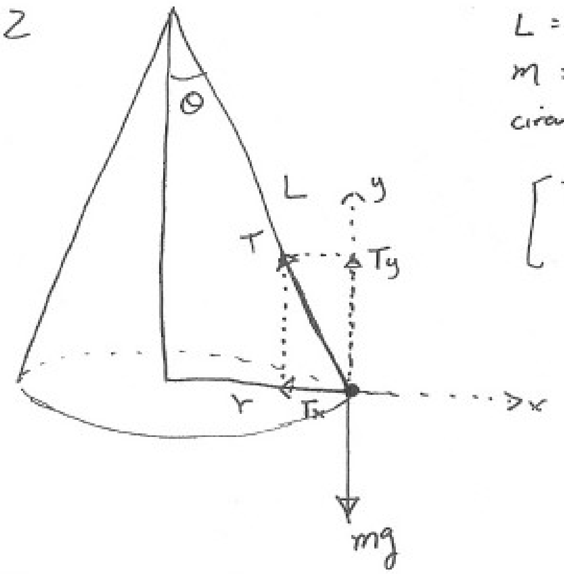
$F_{centri} = T_1 = m\dfrac{v^2}{r}$

This force comes from the weight of M via tension of a cord. Hence $T_1 = T_2$

$m\dfrac{v^2}{r} = Mg$

$\therefore v = \underline{\underline{\sqrt{\dfrac{Mgr}{m}}}}$

#52



$L = 0.90\ m$

$m = 0.040\ kg$

circumference $= 0.94\ m = 2\pi r$

Note:
[T_y cancels mg & T_x is the centripetal force]

x comp

$$-T\sin\theta = F_{centri} = -\frac{mv^2}{r} \quad \text{--- (1)}$$

y comp

$$T\cos\theta - mg = 0 \quad \text{--- (2)}$$

(a) Circumference $= 0.94\ m = 2\pi r$

$$\therefore r = \frac{0.94\ m}{2\pi} \quad \text{--- (3)}$$

Also

$$\theta = \sin^{-1}\frac{r}{L} = \sin^{-1}\frac{\frac{0.94\ m}{2\pi}}{0.9\ m} = 9.568607737^\circ \quad \text{--- (4)}$$

(2) ← (4), solve for T

$$T = \frac{mg}{\cos\theta} = \frac{0.040\ kg \cdot 9.81\ m/sec^2}{\cos(9.5686\ldots^\circ)} = \underline{\underline{0.397936384\ N}}$$

(b) student way : (1) ← (2)′ solve for v

$$V = \sqrt{\frac{T\sin\theta\, r}{m}} = 0.497397456\ m/sec \quad \text{--- (1)}'$$

$$P = \frac{2\pi r}{V} = \frac{2\pi r}{0.49\ldots} = \underline{\underline{1.889836765\ sec}}$$

(Seems easy enough. However, there are so many numbers you must calculate and write them down. If you mistype on a calculator, you have to retype — waste of time!)

Takashi way : $V = \sqrt{\frac{T\sin\theta\, r}{m}}$ & $P = \frac{2\pi r}{V}$

$$\therefore P = \frac{2\pi r}{\sqrt{\frac{T\sin\theta\, r}{m}}} = 2\pi\sqrt{\frac{r^{\cancel{2}} \cdot m}{T\sin\theta\, \cancel{r}}} = 2\pi\sqrt{\frac{r \cdot m}{\frac{mg}{\cos\theta}\sin\theta}} \qquad \left(T = \frac{mg}{\cos\theta}\right)$$

$$= 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{r}{g \cdot \frac{r}{L}}} = 2\pi\sqrt{\frac{L}{g}}$$

The last eqn. does not contain any # calculated in part (A). All givens. The point is "try to reduce as much as possible before you plug in #'s. Also, just for fun check the eqn. 15 – 28 in ch. 15, P.396. We derived the eqn. without knowing it!

ch. 7 #2, 5, 35, 46, 50

#2 (a) $\Delta KE = KE_f - KE_i$

$= \frac{1}{2} m \cancel{V_f^2}^{0} - \frac{1}{2} m V_i^2$

$= -\frac{1}{2} (4\times10^6 \text{ kg})(15000 \text{ m/sec})^2 = \underline{\underline{-4.5\times10^{14} \text{ J}}}$

(b) $4.5\times10^{14} \text{ J} \times \frac{1 \text{ Megaton}}{4.2\times10^{15} \text{ J}} = \underline{\underline{0.107 \text{ megaton bomb}}}$

(c) $4.5\times10^{14} \text{ J} \times \frac{1 \text{ Hiroshima}}{13 \text{ Kiloton}} \times \frac{1000 \text{ Kilo ton}}{1 \text{ Mega ton}} \times \frac{1 \text{ Megaton}}{4.2\times10^{15} \text{ J}} = \underline{\underline{8.24 \text{ Hiroshima bomb.}}}$

#5 let m_f = mass of father
m_s = " son $(= \frac{1}{2} m_f)$
V_f = vel of father
V_s = " son

$KE_f = \frac{1}{2} KE_s$

$\frac{1}{2} m_f V_f^2 = \frac{1}{2} \left(\frac{1}{2} m_s V_s^2\right)$

$\frac{1}{2} m_f V_f^2 = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} m_f\right) V_f^2\right)$

$V_f^2 = \frac{1}{4} V_s^2$ ——— ①

Now the father's speed is $(V_f + 1 \text{ m/sec})$

$KE_f = KE_s$

$\frac{1}{2} m_f (V_f + 1)^2 = \frac{1}{2} m_s V_s^2$

$\frac{1}{2} m_f (V_f + 1)^2 = \frac{1}{2} \left(\frac{1}{2} m_f\right) V_s^2$

$2(V_f^2 + 2V_f + 1) = V_s^2$ ——— ②

① ← ②

$V_f^2 = \frac{1}{4} \left(2 (V_f^2 + 2V_f + 1)\right)$

$V_f^2 - 2V_f - 1 = 0 \rightarrow V_f = \pm 2.41 \text{ m/sec}$ ——— ①′

① ← ①′

$\underline{\underline{V_s = \pm 4.82 \text{ m/sec}}}$ (± indicate their directions)

#35 $W = \int F \cdot dx$

$= \int m \cdot a \cdot dx$

$= \int m \frac{dv}{dt} \cdot dx$

$= \int m \, dv \cdot \frac{dx}{dt}$

$= \int m \cdot dv \cdot \cancel{v}$

$= m \int v \cdot dv$

$= m \int v \cdot \frac{dv}{dt} \cdot dt$

$= m \int v \cdot a \cdot dt$ ——— ①

$X = 3t - 4t^2 + 1t^3$ ——— ②

$\frac{dx}{dt} = v = 3 - 8t + 3t^2$

$\frac{dv}{dt} = a = -8 + 6t$ ——— ③

① ← ② & ③

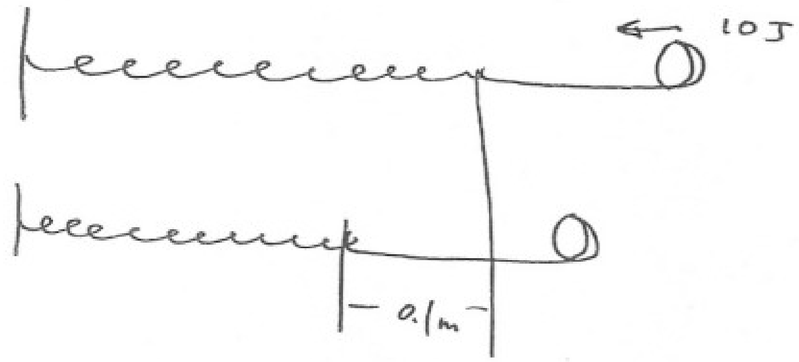
$W = 3 \int_0^4 (3 - 8t + 3t^2)(-8 + 6t) \cdot dt$

$= \int_0^4 (-72 + 246t - 216t^2 + 54t^3) \cdot dt$

$= -72t + 123t^2 - 72t^3 + \frac{54}{4} t^4 \Big|_0^4$

$= \underline{\underline{528\ J}}$

\# 46



$m_{ladle} = 0.3\,kg$

$k = 500\,N/m$

$$P = \frac{dw}{dt} = \frac{d(\int F \cdot dx)}{dt}$$

if F is const,

$$= F\frac{d(\int dx)}{dt}$$

$$= F \cdot \frac{dx}{dt} = \underline{\underline{F \cdot V}}$$

if not

$P = F \cdot V$ is not ~~true~~.

(a) at Equilibrium, $F = 0$

Hence $P = \frac{d\int 0 \cdot dx}{dt} = \underline{\underline{0\ w}}$

(b)

$$P = \frac{d(\int F \cdot dx)}{dt} = \frac{d(\int kx \cdot dx)}{dt}$$

$$= \frac{d \cdot (\frac{1}{2}kx^2)}{dt}$$

$$= \frac{1}{2}k\left(2x \cdot \frac{dx}{dt}\right)$$

$$= \frac{1}{2}k\, 2x \cdot V = kxV$$

$$10\,J = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}(0.3)V^2 + \frac{1}{2}\,500\,N/m \cdot (0.1m)^2$$

$V = 7.071067812\ m/sec$ (at $x = 0.1m$)

So $P = kxV$

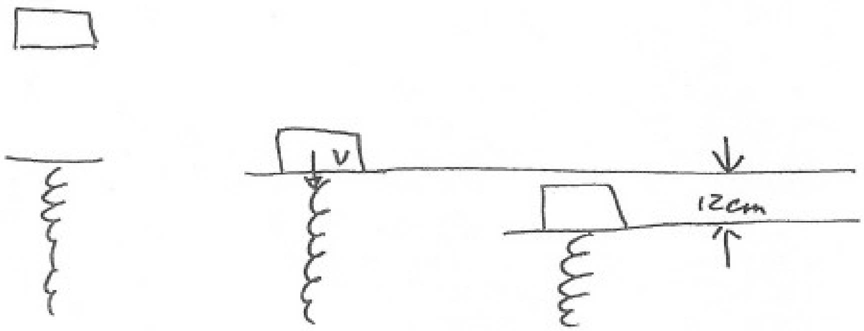
$= 500\,N/sec\ 0.1m \cdot 7.071067812\ m/sec$

$= 353.5533906\ w$

$\therefore$ = 353.55 w spring taking away from the ladle.

50

7-4



$m = 0.25\,kg$

$K = 2.5\,N/cm = 250\,N/m$

$X = 12\,cm = 0.12\,m$

(a) ΔPE

$mgx = 0.25 \cdot 9.81 \cdot 0.12 = \underline{\underline{0.2943\,J}}$

(b) ΔSE

$\frac{1}{2}kx^2 = \frac{1}{2}\ 250\,N/m \cdot (0.12)^2 = \underline{\underline{1.8\,J}}$

the energy was stored into the spring. the spring did a negative work. $\Rightarrow$ $\underline{\underline{-1.8\,J}}$

(c) the spring stored 1.8 J of energy which came from KE & PE.

$\Delta KE + \Delta PE = \Delta SE$

$\frac{1}{2}mV^2 + mgx = \frac{1}{2}kx^2$

$V = \sqrt{\frac{2(\frac{1}{2}kx^2 - mgx)}{m}} = 3.470677167\,m/sec$

$= \underline{\underline{3.47\,m/sec}}$

(d) Once again.

$\Delta KE + \Delta PE = \Delta SE$ $\qquad V = 2 \times 3.470677167\,m/sec$

$\frac{1}{2}mv^2 + mgx = \frac{1}{2}kx^2$

$\frac{1}{2}kx^2 + mgx - \frac{1}{2}mV^2 = 0$

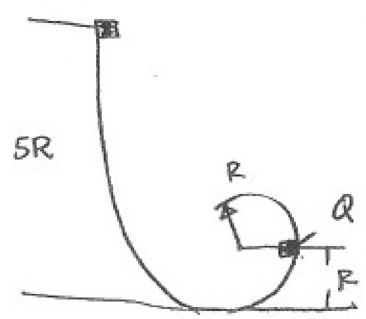
$x = \frac{mg \pm \sqrt{(mg)^2 - 4(\frac{1}{2}k)(\frac{1}{2}mV^2)}}{2(+\frac{1}{2}k)}$

$= 0.229533998\,m$

$= \underline{\underline{22.95\,cm}}$

Ch. 8 #7, 8, 23, 29, 70, 85, 135

#7



(a) work done by gravitational force $= mg\Delta h$

$\therefore \; mg(5R-R)$

$= \underline{\underline{4mgR}}$

b) $mg(5R-2R)$

$= \underline{\underline{3mgR}}$

(c) $\underline{\underline{5mgR}}$

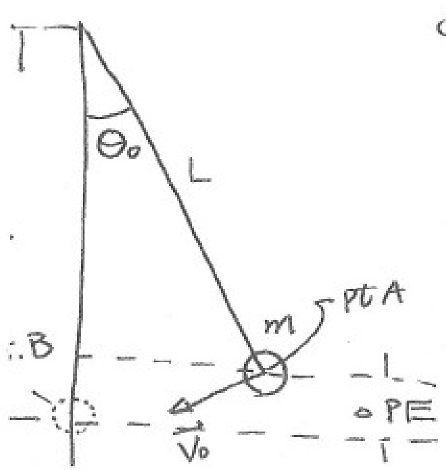
(d) $\underline{\underline{mgR}}$

(e) $\underline{\underline{2mgR}}$

(f) the same since we are ~~talking about~~ gravitational Pot. energy.

Now. Can you answer the net force the block feels at Q and at the top of the loop?

#8.



(a) Lost of ΔPE is Gain of ΔKE.

At Pt. A

$E_{Total} = PE + KE$

$= mg(L - L\cos\theta_0) + \frac{1}{2}mV_0^2$

$= mgL(1-\cos\theta_0) + \frac{1}{2}mV_0^2$

At. B

$E_{Total} = KE$

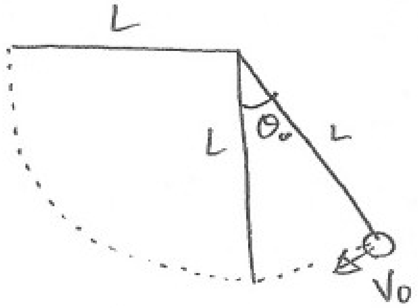
$= \frac{1}{2}mV^2$

Cons. of Energy

$$mgL(1-\cos\theta_0) + \frac{1}{2}mV_0^2 = \frac{1}{2}mV^2$$

$$V = \pm\sqrt{2gL(1-\cos\theta_0) + V_0^2}$$ ($\pm$ shows directions)

b)



$$E_i = E_f$$

$$\frac{1}{2} m V_0^2 + m g L (1-\cos\theta_0) = m g L$$

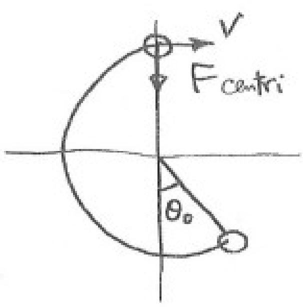
$$\frac{1}{2} V_0^2 = gL - gL(1-\cos\theta_0)$$

$$V_0^2 = 2(gL - gL + gL\cos\theta_0)$$

$$= 2gL\cos\theta_0$$

$$\therefore \quad V = \pm\sqrt{2gL\cos\theta_0}$$

c)



At the top of its motion:
Because the string is straight, we need a centripetal force. The minimum centripetal force needed is its own weight!

$$F_{centri} = m\frac{V^2}{r} = mg$$

$$\Downarrow$$

$$\frac{V^2}{L} = g$$

$$\therefore V = \sqrt{gL}$$ (So, at the top, the block moves at $V = \sqrt{gL}$)

$$E_{Total\ at\ Top} = PE + KE$$

$$= mg(2L) + \frac{1}{2}mV^2$$

$$= 2mgL + \frac{1}{2}m(gL)$$

$$= \frac{5}{2}mgL$$

$$E_i = E_f$$

$$\frac{1}{2}mV_0^2 + mgL(1-\cos\theta_0) = \frac{5}{2}mgL$$

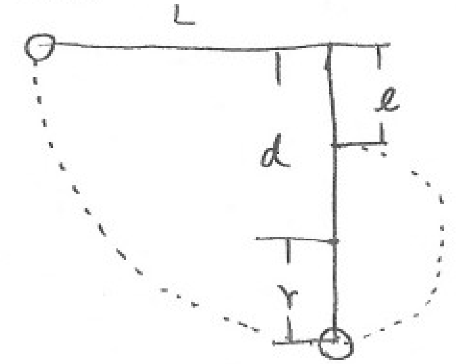
$$V_0^2 = 5gL - 2gL + 2gL\cos\theta_0$$

$$= 3gL + 2gL\cos\theta_0$$

$$V = \sqrt{gL(3 + 2\cos\theta_0)}$$

(d) If θ_0 is greater, the original PE is greater.
Hence the required V_0 is smaller for (b) & (c)

#23



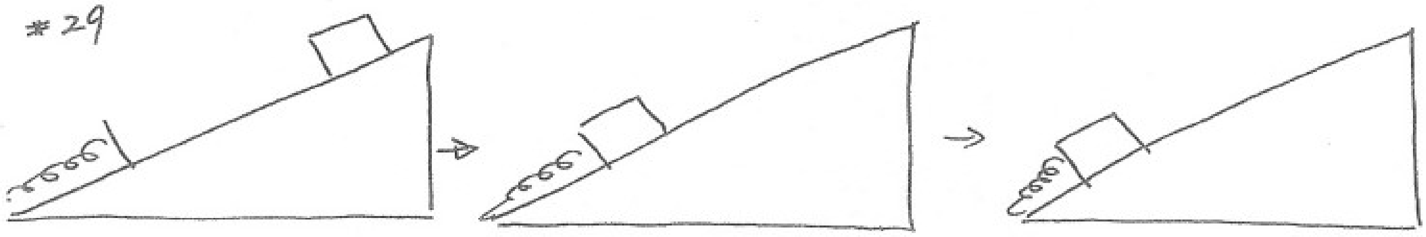
$$E_i = E_f$$
$$PE = KE$$
(a) $$mgL = \frac{1}{2} m V^2$$
$$V = \sqrt{2gL}$$
$$= \underline{4.8522 \text{ m/sec}}$$

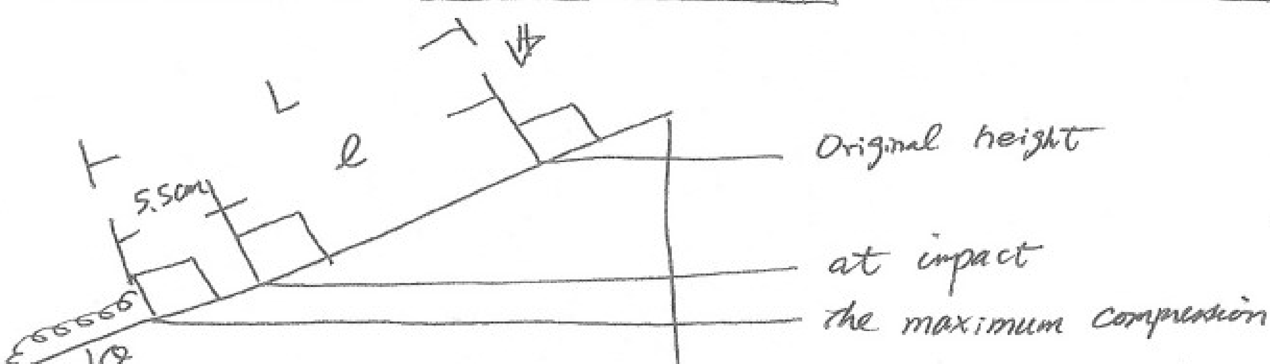
(b) $d = 0.75\,m$ $\quad \therefore r = (1.2 - 0.75)\,m$
$$= 0.45\,m$$

Hence at the top of the smaller circular motion

$$E_i = E_f$$
$$PE_i = PE_f + KE$$
$$mgL = mg(2r) + \frac{1}{2} m V^2$$
$$\frac{1}{2} m V^2 = mg(L - 2r)$$
$$V = \sqrt{2g(L-2r)}$$
$$= \underline{2.426 \text{ m/sec}}$$

#29





The easiest way to think about this problem is that the original P.E. was converted into S.E. at the max. compression. 8-4

(a) $k = \frac{270\,N}{0.02\,m} = 13500\ N/m$

$m = 12\,kg$

$\theta = 30°$

$E_i = E_f$

$mgh = \frac{1}{2}kx^2$

$mgL \sin 30° = \frac{1}{2}(13500\ N/m)(0.055)^2$

$mg(l + 0.055)\sin 30°$

Solve for l

$l = 0.29190\,3669\ m$

So

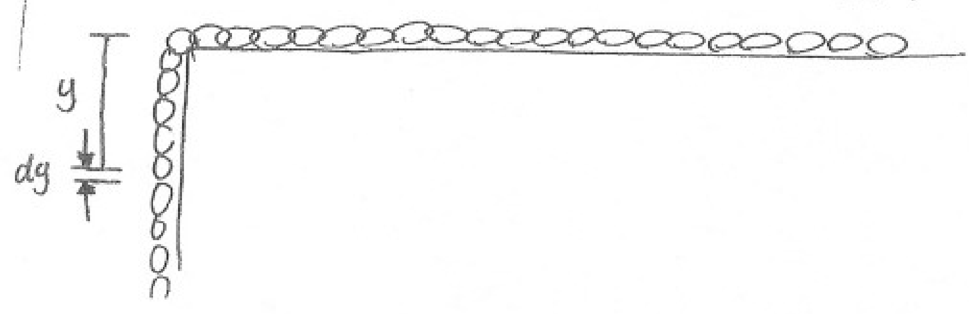
$L = l + 0.055 = \underline{0.3469\ m}$

(b) $mgl \sin 30° = \frac{1}{2}mV^2$

$V = \sqrt{2gl \sin 30°} = \underline{1.6922\ m/sec}$

\#70 Do not think about the <u>whole</u> hanging part. Just like other times I have been stressing, think about a point (In this case, a small piece of the hanging chain). This little piece is "y" distance away from the top and its length is "dy" with its mass "dm". ("d" for anything small.) The relationship between length & mass is;

$dm = \lambda \cdot dy$ where "λ" is a (linear) density of the chain, $\frac{m}{L}$.



the work required to bring "dm" from where it is to the top of the Table is

$$dw = (dm)\, g\, y \quad (\text{just like } w = mgh)$$

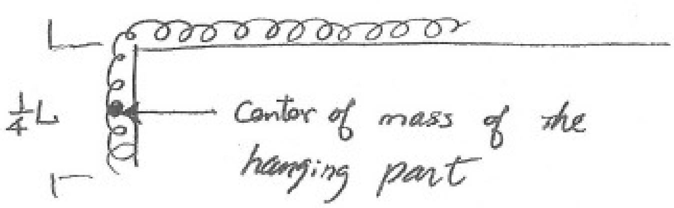
So,

$$dw = (\rho\, dy)\, g\, y \quad (\text{changing from two variables to one variable})$$
$$= \rho g y \cdot dy$$

So the total work is done when y changes from 0 to $\frac{1}{4}L$

$$W = \int dw = \int_0^{\frac{1}{4}L} \rho g y \cdot dy$$
$$= \rho g \tfrac{1}{2} y^2 \Big|_0^{\frac{1}{4}L}$$
$$= \rho g \tfrac{1}{2} \left(\tfrac{1}{4}L\right)^2 = \tfrac{1}{32} \rho g L^2$$
$$= \tfrac{1}{32} \tfrac{m}{L} g L^2 = \underline{\underline{\tfrac{1}{32} mgL}}$$

Being able to think this way is very very important. Later on this semester (and in 231), we have to be able to use this method. (that is why you are taking Calculus). the following way is a solution using pre-calculus.



The total mass of the hanging part $\rightarrow \frac{1}{4}m$

the dist. from the top to the c.m. of the hanging part $\rightarrow \frac{1}{8}L$

$$W = \Delta PE = mg\Delta h = \left(\tfrac{1}{4}m\right) \cdot g \left(\tfrac{1}{8}L\right)$$
$$= \underline{\underline{\tfrac{1}{32} mgL}}$$

This looks easier. However, its almost impossible to solve this type of question with this method if the density of the chain is not constant.

However with calculus, even if the density is not const, as long as the density fcn. is given, we can solve it.

\# 85 $P = 1.5\,MW = 1.5 \times 10^6\,w$

$V_i = 10\,m/sec$ (at $t_i = 0\,sec$)

$V_f = 25\,m/sec$ (at $t_f = 360\,sec$)

This is a greate problem to check your understanding in:

- Force & Work relation
- Work - Energy theorem
- Work - Power relation

You should be able to explain them in writing (Not eqns only)

Also, students usually assume the force is const., but the problem does not state so. It is safe to assume it is not so if it is not stated so. However, P is const. in this case.

(a)

$$W = \int F \cdot dx = \int P \cdot dt$$

$$\int ma \cdot dx = \bar{P} \cdot t \quad (P \text{ is const. in this case})$$

$$\int m \frac{dv}{dt} \cdot dx = \bar{P} \cdot t$$

$$\int m \frac{dx}{dt} \cdot dv = \bar{P} \cdot t$$

$$\int m\, v \cdot dv = \bar{P} \cdot t$$

$$\frac{1}{2} m v^2 \Big|_{V_i}^{V_f} = \bar{P} \cdot t$$

$$\therefore \bar{P} \cdot t_f = \frac{1}{2} m \left(V_f^2 - V_i^2\right) \quad \text{———— (1)}$$

$$\therefore m = \frac{2\bar{P} t_f}{V_f^2 - V_i^2} = \frac{2 \cdot 1.5 \times 10^6\,w \cdot 360\,sec}{(25\,m/sec)^2 - (10\,m/sec)^2}$$

$$= \underline{\underline{2.057 \times 10^6\,kg \quad (2057142.857\,kg)}}$$

(b) Instead of using t_f in eqn. (1), we can use $0 \le t \le 6\,min$

$$\therefore \bar{P} t = \frac{1}{2} m \left(V^2 - V_i^2\right)$$

$$\therefore V = \sqrt{\frac{2\bar{P} t}{m} + V_i^2} \quad \text{———— (2)}$$

$$= \sqrt{\frac{2\bar{P} t}{\frac{2\bar{P} t_f}{V_f^2 - V_i^2}} + V_i^2}$$

$$= \sqrt{(V_f^2 - V_i^2)\frac{t}{t_f} + V_i^2}$$

$$= \sqrt{\frac{((25\,m/sec)^2 - (10\,m/sec)^2)}{360\,sec}\cdot t + (10\,m/sec)^2}$$

$$= \underline{\underline{\sqrt{1.458\dot{3}\,t + 100}\ \ m/sec}}$$

(c) Using eqn ②.

$$a = \frac{dv}{dt} = \frac{1}{2}\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{-\frac{1}{2}}\frac{2P}{m}$$

$$= \frac{P}{m}\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{-\frac{1}{2}}$$

$$\therefore F = ma = P\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{-\frac{1}{2}}$$

$$= \underline{\underline{\frac{1.5\times 10^6}{\sqrt{1.458\dot{3}t + 100}}\ N}}$$

(d)

$$X = \int_0^{360} v\cdot dt$$

$$= \int\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{1/2}\cdot dt$$

$$= \frac{m}{2\bar{P}}\cdot\frac{2}{3}\left(\frac{2\bar{P}t}{m} + V_i^2\right)^{3/2}\Big|_0^{360}$$

$$= \frac{m}{3\bar{P}}\left(\frac{2\bar{P}}{m}t + V_i^2\right)^{3/2}\Big|_0^{360}$$

$$= \frac{2\bar{P}t_f}{3\bar{P}(V_f^2 - V_i^2)}\left(1.458\dot{3}t + 100\right)^{3/2}\Big|_0^{360}$$

$$= \frac{2\cdot 360}{3(25^2 - 10^2)}\left[(1.458\dot{3}\cdot 360 + 100)^{3/2} - (100)\right]$$

$$= 6.68771523\times 10^3\,m$$

$$= \underline{\underline{6.69\times 10^3\,m}}$$

Work – Energy theorem.

A positive work done to a system causes the system to gain K.E. (a negative work can be done as well)

$$W = \Delta K.E.$$

Force – Work Relation

To change KE ($= \frac{1}{2} m v^2$), acceleration is needed which, of course, is caused by an outside force. Hence, outside force causes work. (Positive Force onto the system causes positive work done to the system) That's why this is the dot product.

$$W = \int \vec{F} \cdot d\vec{x} = \frac{1}{2} m (v_f^2 - v_i^2) = \Delta KE$$

Work – Power Relation

Power is a rate of change of work done w.R.T. time

$$P = \frac{dw}{dt}$$

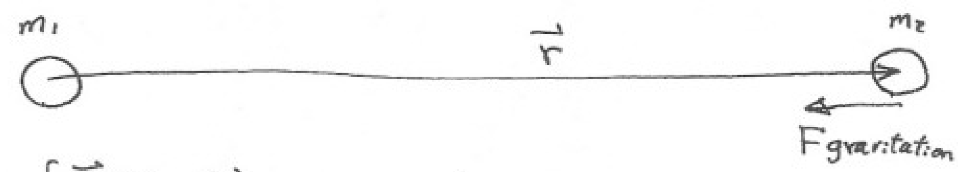
Over all

$W = \Delta KE$ can be calculated by two ways

$$W = \int \vec{F} \cdot d\vec{x} = \int P \cdot dt$$

A typical mistake students make is that they mix the variables. Integrate $\underline{F}$ w.R.T. $\underline{\text{distance}}$ and $\underline{P}$ w.R.T. $\underline{\text{time}}$. You can see if you don't practice, you can be easily confused. – this is the difference between Math & Physics. Physics does not tell you integrate what w.R.T. what. Make sure you know the relations.

\# 135



(a)

$$W = \int \vec{F}(r) \cdot d\vec{r}$$

$$= -\int_{\infty}^{R} G \frac{m_1 m_2}{r^2} \cdot d\vec{r}$$

$$= G m_1 m_2 \left. \frac{1}{r} \right|_{\infty}^{R}$$

$$= G m_1 m_2 \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

$$= \frac{G m_1 m_2}{R}$$

(I hate to use X in space, there is no horizontal direction. Also, forces are the same if the distances are the same for any direction)

$$\vec{F} \cdot \vec{r} = |\vec{F}||\vec{r}| \cos 180^\circ \text{ (they are opposite)}$$

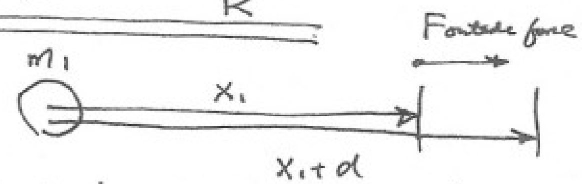
$$= -|\vec{F}||\vec{r}|$$

$$PE(r \to \infty) = 0$$

This shows that the gravitational work did + work onto m_2 if brought ∞ to R. $\Delta KE = -\Delta PE$

$$\therefore \quad PE = -\frac{G m_1 m_2}{R}$$

(b)



Outside force to bring m_2 from x_1 to $x_1 + d$ does a positive work
(Not Gravitational Force)
(Gain of energy)

$$W = \int \vec{F}(r) \cdot d\vec{r}$$

This case $\vec{F}$ & $\vec{r}$ are the same direction.

$$= \int_{x_1}^{x_1+d} G \frac{m_1 m_2}{r^2} \cdot dr$$

$$= G m_1 m_2 \left. -\frac{1}{r} \right|_{x_1}^{x_1+d}$$

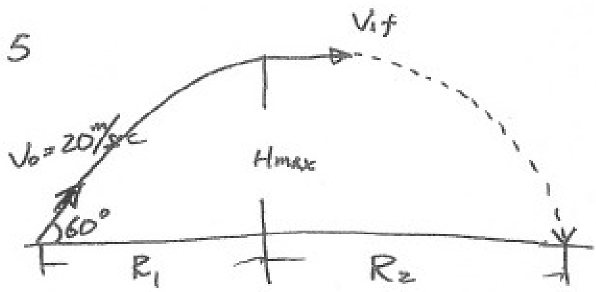
$$= G m_1 m_2 \left(\frac{1}{x_1} - \frac{1}{x_1 + d} \right)$$

$$= G m_1 m_2 \left(\frac{(x_1 + d) - x_1}{x_1 (x_1 + d)} \right)$$

$$= G m_1 m_2 \frac{d}{x_1 (x_1 + d)}$$

ch. 9. #15, 17, 20, 27, 49, 85, 112, 130

15



Eqns. until the shell reaches its max (H_{max})

x

$a_x = 0$ —— ①
$V_x = V_0 \cos 60°$ —— ②
$X = V_0 t \cos 60°$ —— ③

y

$a_y = -g$ —— ④
$V_y = -gt + V_0 \sin 60°$ —— ⑤
$y = -\frac{1}{2} g t^2 + V_0 t \sin 60°$ —— ⑥

For H_{max}.

Eqn ⑤ $V_y = 0$ solve for t

$0 = -gt + V_0 \sin 60°$
$gt = V_0 \sin 60°$
$t = \dfrac{V_0 \sin 60°}{g}$ —— ⑤'

⑥ ← ⑤'

$$y = -\frac{1}{2} g \left(\frac{V_0 \sin 60°}{g}\right)^2 + V_0 \left(\frac{V_0 \sin 60°}{g}\right) \sin 60°$$
$$= \frac{1}{2}\left(\frac{V_0^2 \sin^2 60°}{g}\right) \equiv H_{max} = 15.29051988 \, m$$

③ ← ⑤'

$$X = V_0 \left(\frac{V_0 \sin 60°}{g}\right) \cos 60° \equiv R_1 = 17.65597153 m$$

Eqns. after it breaks into two parts

p_{ix} $\qquad$ p_f

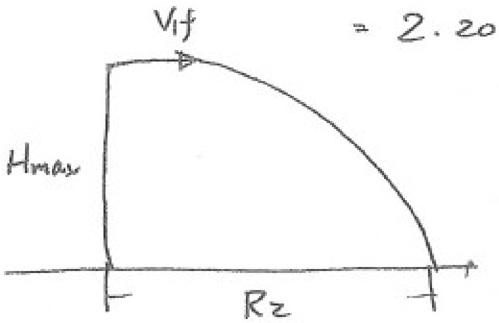
$$m_i V_{ix} = m_1 V_{1f} + m_2 \cancel{V_{2f}}^{0}$$

$\Downarrow$ $\quad m_1 = m_2 = \frac{1}{2} m_i \equiv m \quad$ $\Downarrow$

$$2m V_{ix} = m V_{1f}$$

$\therefore V_{1f} = 2 V_{ix}$

$$V_{1f} = 2 V_0 \cos 60°$$



$$= 2 \cdot 20 \cdot \tfrac{1}{2} = 20\ \text{m/sec}$$

x

$a_x = 0$ — ①

$V_x = 20\ \text{m/sec}$ — ②

$X = 20t$ — ③

y

$a_y = -g$ — ④

$V_y = -gt + \cancel{V_{0y}}^{\,0}$ — ⑤

$y = -\tfrac{1}{2} g t^2 + \cancel{y_0}^{\,H_{max}}$ — ⑥

to calculate R_2 :

Eqn. ⑥ $y = 0$, solve for t

$$0 = -\tfrac{1}{2} g t^2 + H_{max}$$

$$\tfrac{1}{2} g t^2 = H_{max}$$

$$t = \sqrt{\frac{2H_{max}}{g}} \quad \text{— ⑥}'$$

③ ← ⑥′

$$X = 20\sqrt{\frac{2H_{max}}{g}} = 35.31194307\ m$$

Range $= R_1 + R_2 = 52.97\ m$ $(52.96791460\ m)$

This ans. is exactly $3R_1$ meaning $R_2 = 2R_1$.

Can you justify why? (You should be able to .. it is an important conceptual question.)

Also, if this event happend on another place,

$$R_1 = \frac{2V_0^2 \sin 60° \cos 60°}{a}$$

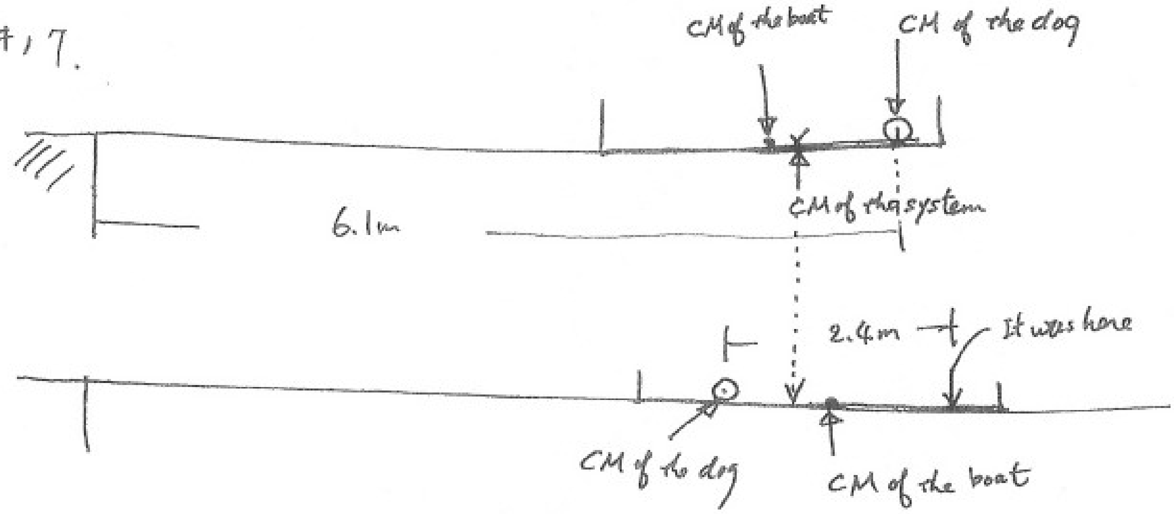
$$R_2 = \frac{2V_0^2 \sin 60° \cos 60°}{a}$$

$$\therefore R_T = R_1 + R_2 = \frac{3V_0^2 \sin 60° \cos 60°}{a} \quad (\text{still } 3R_1)$$

$$= \frac{3\sqrt{3}}{4} \frac{V_0^2}{a}$$

(where a is a gravitational acc. of the planet)

#17.



CM of the system will not move. Let x be the distance that CM of the boat moved (away from the shore). Then, $(2.4-x)$ is the distance that the dog moved closer to the shore.

$$m_b d_b = m_d d_d$$

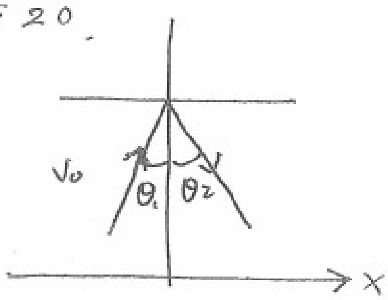
$$18\ x = 4.5\ (2.4-x)$$

$$x = 0.48\ m$$

So the doggie is $2.4 - 0.48 = 1.92\ m$ closer to the shore.

$$6.1 - 1.92 = \underline{\underline{4.18\,m}}$$

#20.



$\theta_1 = 30°$

(a) cons. of p (in x direction)

$$\underset{p_{ix}}{m V_0 \sin\theta_1} = \underset{p_{fx}}{m V_f \sin\theta_2} \quad (V_0 = V_f)$$

$$\therefore\ \theta_1 = \theta_2 = \underline{\underline{30°}}$$

(b) $\Delta p_y = p_{fy} - p_{iy}$

$= -mV_0 \cos 30°$ (down) $- mV_0 \cos 30°$ (up)

$= -2mV_0 \cos 30°$

$= -0.5715767665\ \mathrm{kg\,m/sec}$ ((−) means downward)

#27 Impulse = $\Delta p = \int F \cdot dt = (mV_f - mV_i)$

$\therefore F = \frac{dp}{dt}$ (rate of change of p)

Impulse given to a bullet (by Superman)

$\Delta p = m_b V_{bf} - m_b V_{bi} = 2 m_b V_{bf}$ (since $V_{bf} = -V_{bi}$)

Total impulse given to bullets per second

$\frac{\Delta p_T}{sec} = \#\ \text{bullets}/sec \cdot 2 m_b V_{bf}$

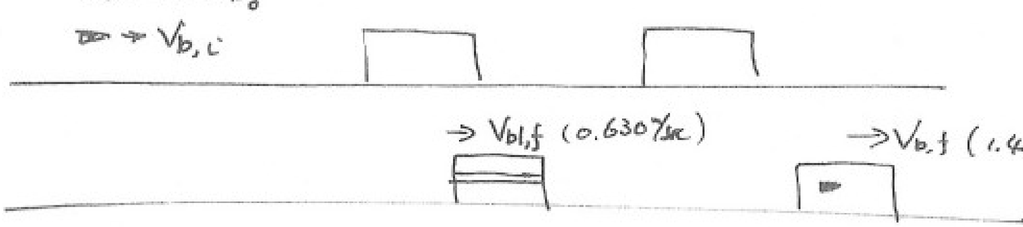
$= \frac{100 \text{ bullets}}{60 \text{ sec}} \cdot 2(0.003 kg)(-500 \text{ m/sec})$

$= -5N$ (Force given to opposite direction of original direction of bullets)

By Newton's third law, if we take Superman & bullets as a system, there is no net force. Superman must take equal & opposite reaction.

$\therefore \underline{F_{\text{onto superman}} = 5N}$

#49. $m_b = 0.0035kg$



(a) take p_i as the bullet between the blocks (We don't need to worry about p of m_1 since there is no change during the collision of the bullet and m_2)

p_i $\qquad\qquad$ p_f

$m_b V_{b,\text{between}} = (m_b + m_2) V_{b,f}$ (perfectly inelastic collision)

$V_{b,\text{between}} = \frac{m_b + m_2}{m_b} V_{b,f}$

$= \left(\frac{0.0035kg + 1.8kg}{0.0035kg}\right)(1.4 \text{ m/sec}) = \underline{721.4 \text{ m/sec}}$

(b) take p_i as the bullet before it hits m_1

p_f as the bullet embedded in block m_2

$\underline{p_i}$

Don't forget this.

$$m_b V_{b,i} = m_1 V_{b1,f} + (m_b + m_2) V_{b2,f}$$

$$V_{bi} = \frac{m_1}{m_b} V_{b1,f} + \frac{m_b + m_2}{m_b} V_{b2,f}$$

$$= 216 \text{ m/sec} + 721.4 \text{ m/sec} = \underline{\underline{937.4 \text{ m/sec}}}$$

#85

→ 7600 m/sec

290 kg	150 kg

m_R : mass of the rocket

m_C : " capsule

V_i : 7600 m/sec

Since there is no external force, p conserves.

p_i $\qquad\qquad$ p_f

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C V_{cf}$$

Also their relative speed is 910 m/sec

$$\rightarrow V_{cf} = V_{Rf} + 910$$

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C (V_{Rf} + 910)$$

$$(m_R + m_C) V_i = m_R V_{Rf} + m_C V_{Rf} + 910 m_C$$

$$(m_R + m_C) V_i - 910 m_C = (m_R + m_C) V_{Rf}$$

$$V_{Rf} = \frac{(m_R + m_C) V_i - 910 m_C}{m_R + m_C}$$

$$= \underline{\underline{7289.772727 \text{ m/sec}}}$$

(a) & (b)

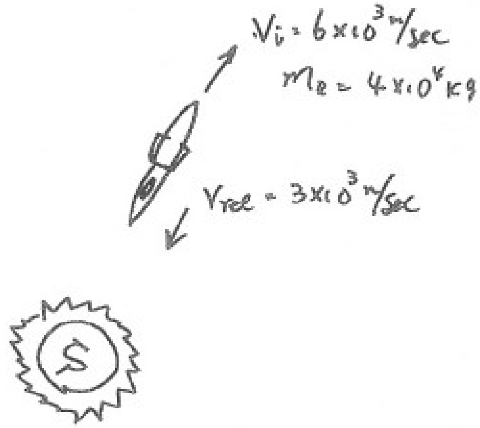
$$V_c = V_{Rf} + 910 = \underline{\underline{8199.772727 \text{ m/sec}}}$$

(C) & (d) $KE_i = \frac{1}{2}(m_R + m_C) V_i^2 = \underline{\underline{1.27072 \times 10^{10}\ J}}$

$KE_f = \frac{1}{2} m_R V_{Rf}^2 + \frac{1}{2} m_C V_{Cf}^2 = \underline{\underline{1.274813449 \times 10^{10}\ J}}$

why $KE_i < KE_f$? the ΔE came from the spring energy transferred into KE.

\# 112



(a) $F = ma$

$= 4\times10^4\,kg \cdot 2\,m/sec^2$

$= \underline{\underline{8\times10^4\,N}}$

(b) this force ($8\times10^4 N$) must come from the fuel.

Force per second:

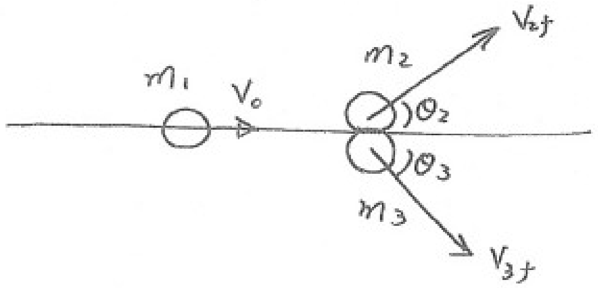
$$\frac{dp}{dt} = \frac{d(mv)}{dt} = \overbrace{V_{rel}\frac{dm}{dt}}^{\text{Fuel}} + \overbrace{\boxed{m\frac{dv}{dt}}}^{\text{Rocket}}$$

equal & opposite.

$$8\times10^4\,N = 3\times10^3\,m/sec\,\frac{dm}{dt}$$

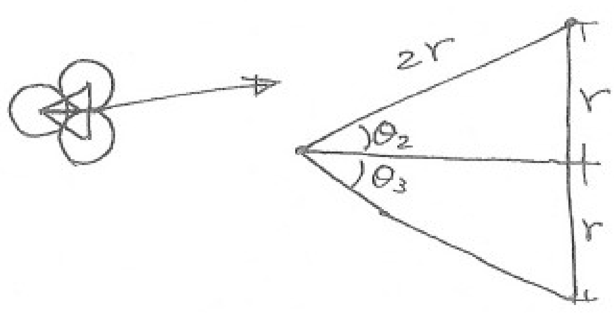
$$\frac{dm}{dt} = \frac{8\times10^4\,N}{3\times10^3\,m/sec} = \underline{\underline{2.\dot{6}\times10^1\,kg/sec}}$$

\# 130



$V_0 = 10$ m/sec

$m_1 = m_2 = m_3 \equiv m$



$\sin^{-1}\frac{r}{2r} = \theta_2 = \theta_3$

$\Rightarrow \theta_2 = \theta_3 = 30° \equiv \theta$

Cons. of p

x

p_{ix}

$m V_0 = m V_{1f} + m V_{2f} \cos\theta + m V_3 \cos\theta$

$V_0 = V_{1f} + V_{2f} \cos\theta + V_3 \cos\theta$ ——— ①

y

$p_{iy} = p_{fy}$

$0 = m V_{2f} \sin\theta - m V_{3f} \sin\theta$

$m V_{2f} \sin\theta = m V_{3f} \sin\theta$

$\therefore V_{2f} = V_{3f} = V_f$ —— ②

① ← ②

$V_0 = V_{1f} + V_f \cos\theta + V_f \cos\theta$

$= V_{1f} + 2 V_f \cos\theta$

$\therefore V_0 = V_{1f} + \sqrt{3}\, V_f$ ——— ①'

Cons. of E

E_i

$\frac{1}{2} m V_0^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m V_{2f}^2 + \frac{1}{2} m V_{3f}^2$

$V_0^2 = V_{1f} + V_{2f}^2 + V_{3f}^2$ (But $V_{2f} = V_{3f} = V_f$)

$V_0^2 = V_{1f} + 2 V_f^2$ ——— ③

Eqn. ①'. solve for V_{1f}

$V_{1f} = V_0 - \sqrt{3}\, V_f$ —— ①'' → ③

$V_0^2 = (V_0 - \sqrt{3} V_f)^2 + 2V_f^2$

$V_0^2 = V_0^2 - 2\sqrt{3}\, V_0 V_f + 3 V_f^2 + 2 V_f^2$

$0 = -2\sqrt{3}\, V_0 V_f + 5 V_f^2$

$5 V_f^2 = 2\sqrt{3}\, V_0 V_f$

$V_f = \frac{2\sqrt{3}}{5} V_0$

(one possible sol. is $V_f = 0$
then $V_0 = V_{1f} = 10$ m/sec
→ as if m_1 went through ... but
that is not what we want)

$= \frac{2\sqrt{3}}{5} \cdot 10 = \underline{6.928203230\ m/sec}$ — ③'

①' ← ③'

$V_{1f} = V_0 - \sqrt{3}\, V_f = \underline{-2\ m/sec}$

$V_{1f} = -2\ m/sec$ (moves backward)
$V_{2f} = V_{3f} = 6.928\ m/sec$ at $\pm 30°$

Additional problems: The topics put presented in this chapter used be presented in two chapters. There are more important problems to check your understanding. Make sure you do these problems as well as assigned homework questions.

#1 A railroad flatcar of weight W can roll without friction along a straight horizontal track. Initially, a man of weight w is standing on the car, which is moving to the right with sped v_0. What is the change in velocity of the car if the man runs to the left so that his speed relative to the car is v_{rel}?

#2 An 8.0 kg body is traveling at 2.0 m/sec with no external force acting on it. At a certain instant, an internal explosion occurs, splitting the body into two chunks of 4.0 kg mass each. The explosion gives that chunks an additional 16 J of kinetic energy. Neither chunk leaves the ling of original motion. Determine the speed and direction of motion of each of the chunks after the explosion.

#3 A 1500 kg automobile starts from rest on a horizontal road and gains a speed of 72 km/hr in 30 seconds. (a) What is the kinetic energy of the auto at the end of the 30 sec? (b) What is the average power required of the car during the 30 sec interval? (c) What is the instantaneous power at the end of the 30 sec interval, assuming the acceleration is constant?

#4 During a violent thunderstorm, hail of diameter 1.0 cm fall directly downward at a speed of 25 m/sec. There are estimated to be 120 hails per cubic meter of air. (a) What is the mass of each hailstone (density 0.92 g/cm^3)? (b) Assuming that the hail does not bounce, find the magnitude of the average force on a float room measuring 10 m X 20 m due onto the impact of the hail. (Hint: During impact, the force on a hailstone from the roof is approximately equal to the net force on the hailstone, because the gravitational force on it is small.)

#5 A ball having a mass of 150 g strikes a wall with a speed of 5.2 m/sec and rebounds with only 50% of its initial kinetic energy. (a) What is the speed of the ball immediately after rebounding? (b) What is the magnitude of the impulse on the wall from the ball? (c) If the ball was in contact with the wall for 7.6 ms, what was the magnitude of the average force on the ball from the wall during this time interval?

#6 Spacecraft Voyager 2 (of mass m and speed v relative to the Sun) approaches the planet Jupiter (of mass M and speed of V_J relative to the Sun). The spacecraft rounds the planet and departs in the opposite direction. What is its speed, relative to the Sun, after this slingshot encounter, which can be analyzed as a collision? Assume v = 12 km/sec and V_J = 13 km/sec (the orbital speed of Jupiter). The mass of Jupiter is much greater than the mass of the spacecraft (M>>m).

Solutions:

#1

W = weight of the car
w = weight of the man
v_0 = original velocity (ground velocity)
v_c = Final velocity of the car (ground velocity)
v_m = Final velocity of the man(ground velocity)

$$\vec{p}_i \qquad\qquad \vec{p}_f$$

$$\left(\frac{W+w}{g}\right)\vec{v}_0 = \left(\frac{W}{g}\right)\vec{v}_c + \left(\frac{w}{g}\right)\vec{v}_m \quad \text{but } \vec{v}_m = \vec{v}_c + \vec{v}_{rel}$$

$$\left(\frac{W+w}{g}\right)\vec{v}_0 = \left(\frac{W}{g}\right)\vec{v}_c + \left(\frac{w}{g}\right)(\vec{v}_c + \vec{v}_{rel})$$

$$(W+w)\vec{v}_0 = (W)\vec{v}_c + (w)(\vec{v}_c + \vec{v}_{rel})$$

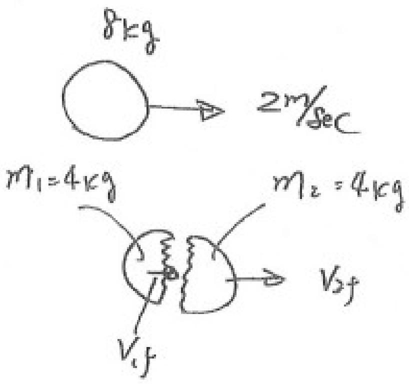
$$(W+w)\vec{v}_0 = (W+w)\vec{v}_c + (w)\vec{v}_{rel}, \text{ and } \Delta\vec{v} = \vec{v}_c - \vec{v}_0$$

$$(W+w)\vec{v}_c - (W+w)\vec{v}_o = -w\vec{v}_{rel}$$

$$(W+w)(\vec{v}_c - \vec{v}_0) = -w\vec{v}_{rel}$$

$$\therefore \vec{v}_c - \vec{v}_0 = -\frac{w\vec{v}_{rel}}{W+w} = \Delta\vec{v}$$

#2



$KE_i = \frac{1}{2} m V_i^2 = 16\text{J}$

from explosion

$KE_f = KE_i + 16\text{J} = 32\text{J}$

Cons. of p

$$(m_1 + m_2) V_i = m_1 V_{1f} + m_2 V_{2f}$$

$$8 \cdot 2 = 4 V_{1f} + 4 V_{2f}$$

$$4 = V_{1f} + V_{2f}$$

$$\therefore V_{1f} = 4 - V_{2f} \quad \text{—— (1)}$$

Cons. of E

$$KE_f = KE_i + 16\text{J}$$

$$\frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 = \frac{1}{2}(m_1 + m_2) V_i^2 + 16\text{J}$$

$$\frac{1}{2}\cdot 4\cdot V_{1f}^2 + \frac{1}{2}\cdot 4\cdot V_{2f}^2 = \frac{1}{2}(.8)2^2 + 16J$$

$$2V_{1f}^2 + 2V_{2f}^2 = 32$$

$$V_{1f}^2 + V_{2f}^2 = 16 \quad \text{———} \quad (2)$$

$(1) \leftarrow (2)$

$$(4 - V_{2f})^2 + V_{2f}^2 = 16$$

$$16 - 8V_{2f} + V_{2f}^2 + V_{2f}^2 = 16$$

$$2V_{2f}^2 - 8V_{2f} = 0$$

$$2V_{2f}(V_{2f} - 4) = 0$$

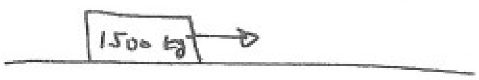
$$V_{2f} = 0 \text{ or } 4\ \text{m/sec}$$

$$V_{1f} = 4\ \text{m/sec or } 0\ \text{m/sec}$$

$\Rightarrow$ One part becomes 0 m/sec & the other 4 m/sec

$$72\ \text{km/hr} = 72000\ \text{m}/3600\ \text{sec} = 20\ \text{m/sec}$$

#3



(a) $KE_f = \frac{1}{2}mV_f^2 = \frac{1}{2}(1500)(20)^2 = \underline{\underline{3.0\times10^5\ J}}$

(b) $\bar{P} = \frac{W}{t} = \frac{3.0\times10^5\ J}{30\ \text{sec}} = \underline{\underline{1.0\times10^4\ \text{watts}}}$

(c) $P = \frac{d(\int F\cdot dr)}{dt} = \frac{d(\int ma\, dr)}{dt}$

since a is const.,

$$= \frac{d(ma\int dr)}{dt}$$

$$= \frac{d(mar)}{dt}$$

$$= ma\frac{dr}{dt}$$

$$= maV$$

a = const

$V = at + \cancel{V_0}^{\,0}$

$20\ \text{m/sec} = a \cdot 30\ \text{sec}$

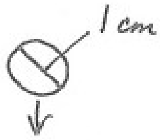
$a = \frac{2}{3}\ \text{m/sec}^2$

$$= 1500 \cdot \frac{2}{3} \cdot 20$$

$$= 2 \times 10^4 \text{ watts}$$

make sure that you know the difference between $\overline{P}$ (average power) and P (instantaneous power).

#4



$D = 1\text{ cm} \rightarrow r = 0.5\text{ cm}$

$V_i = -25\text{ m/sec}$ (down)

(a)

$$m = \rho \cdot \text{vol}$$

$$= 0.92\text{ g/cm}^3 \cdot \frac{4}{3}\pi(0.5\text{ cm})^3 = 0.481710873 5\text{ g}$$

(b)

$$\Delta p_{\text{(each hail)}} = p_f - p_i$$

$$= 0 - mV_i$$

$$= -(0.4817\text{ g})(\overset{\text{down}}{-25\text{ m/sec}})$$

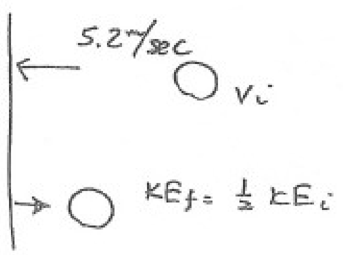
$$= 1.204277184 \times 10^{-2}\text{ kg m/sec} \quad \text{①}$$

How many hail stones per second do they hit the ground?

$$120\text{ stones/m}^3 \cdot 10\text{ m} \times 20\text{ m} \times 25\text{ m} = 6 \times 10^5 \text{ hails} \quad \text{②}$$

$$\overline{F} = \Delta p/\text{hail} \times \frac{\#\text{ hails}}{\text{sec}} = \text{①} \times \text{②} = 7.225663104 \times 10^3\text{ N}$$

#5



(a)

$$KE_i = \frac{1}{2} m_b V_{ib}^2$$

&

$$KE_f = \frac{1}{2} KE_i = \frac{1}{2}\left(\frac{1}{2} m_b V_{ib}^2\right) = \frac{1}{2} m_b V_f^2$$

$$V_f = \sqrt{\frac{1}{2} V_{ib}^2} = 3.676955262\text{ m/sec}$$

(b)

$$\Delta \vec{p} = m_b \vec{V_{bf}} - m_b \vec{V_{bi}}$$

$$= m_b (V_{bf} - V_{bi})$$

It was going "negative" direction.

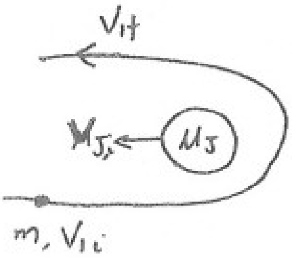
$$= m_b (3.676955\ldots - (-5.2))$$

$$= \underline{\underline{1.33154\,3289 \ \mathrm{kg\,m/sec}}}$$

(c)

$$F = \frac{\Delta p}{\Delta t} = \frac{1.3315 \ \mathrm{kg\,m/sec}}{7.6 \times 10^{-3} \ \mathrm{sec}} = \underline{\underline{175.203644 \ \mathrm{N}}}$$

#6



$V_{1i} = 12 \ \mathrm{km/sec}$

$V_{Ji} = 13 \ \mathrm{km/sec}$

Collision w/o actual physical collision

→ perfectly elastic collision

(as we did in the lab w/ magnets)

⇒ Both p & E conserve.

$\underline{p}$ (I will use the direction of Jupiter as +)

$$\overbrace{M_J V_{Ji} - m V_{1i}}^{p_i} = \overbrace{M_J V_{Jf} + m V_{1f}}^{p_f}$$

$$V_{Jf} = \frac{M_J V_{Ji} - m V_{1i} - m V_{1f}}{M_J}$$

$$= V_{Ji} - \frac{m}{M_J} (V_{1i} + V_{1f})$$

For $m <<<< M_J$ $\quad (m \sim 10^{2\text{-}4} \mathrm{kg},\ M_J \sim 10^{26} \mathrm{kg})$

$\sim V_{Ji}$ ⇒ this indicates that speed of Jupiter will not change.

Also,

$$M_J (V_{Ji} - V_{Jf}) = m (V_{1i} + V_{1f}) \quad \text{———} \quad (1)$$

E

$$\frac{1}{2} M_J V_{Ji}^2 + \frac{1}{2} m V_{1i}^2 = \frac{1}{2} M_J V_{Jf}^2 + \frac{1}{2} m V_{1f}^2$$

$$M_J V_{Ji}^2 + m V_{1i}^2 = M_J V_{Jf}^2 + m V_{1f}^2$$

$$M_J (V_{Ji}^2 - V_{Jf}^2) = m (V_{1f}^2 - V_{1i}^2)$$

$$\underbrace{M_J (V_{Ji} - V_{Jf})}_{\uparrow \; 1}(V_{Ji} + V_{Jf}) = m (V_{1f}^2 - V_{1i}^2)$$

$$m (V_{1i} + V_{1f})(V_{Ji} + V_{Jf}) = m (V_{1f}^2 - V_{1i}^2) = m (V_{1f} + V_{1i})(V_{1f} - V_{1i})$$

$$V_{Ji} + V_{Jf} = V_{1f} - V_{1i}$$

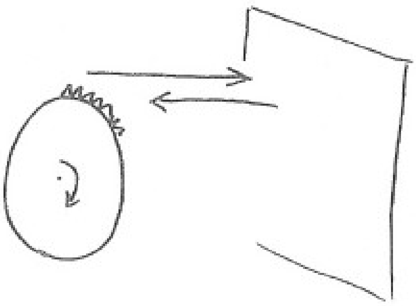
$$\Rightarrow \quad V_{1f} = V_{Ji} + V_{Jf} + V_{1i} = 2 V_{Ji} + V_{1i}$$

$$= 2 (13 \text{ km/sec})(12 \text{ km/sec}) = \underline{38 \text{ km/sec}}$$

(217% increase w/o using fuel!)

Ch. 10 29, 41, 55, 67, 78, 85

#29



The total distance light travels $= 2 \times 500\,m = 1000\,m$

the time for " $= \frac{d}{v} = \frac{1000\,m}{3 \times 10^8\,m/sec} = 3.\dot{3} \times 10^{-6}$ sec

within this time, the wheel must rotate 1 slot out of 500 slots/wheel → How many radian is this?

500 slots = 2π rad

1 slot = ?

1 slot $= \frac{2\pi\,rad}{500} = \underline{1.256637061 \times 10^{-2}\,rad}$

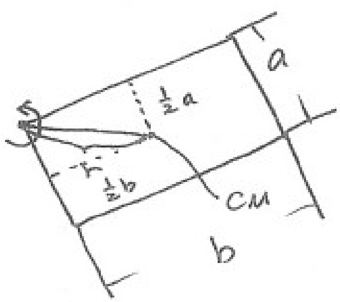
the wheel rotates 1.257×10^{-2} rad in $3.\dot{3} \times 10^{-6}$ sec.

(a) $\omega = \frac{\theta}{t} = \frac{1.257 \times 10^{-2}\,rad}{3.\dot{3} \times 10^{-6}\,sec} = \underline{\underline{3.769911184 \times 10^{3}\,rad/sec}}$

(b) $V = r\omega$

$= 0.05 \times 3.77 \times 10^{3} = \underline{\underline{188.4955592\,m/sec}}$

41



$I = I_{CM} + I_{\text{Rectangle about its center}}$

$r = \left((\tfrac{1}{2}a)^2 + (\tfrac{1}{2}b)^2\right)^{1/2}$

$= \frac{1}{2}(a^2+b^2)^{1/2}$

$I_{CM} = Mr^2 = M\left(\frac{1}{2}(a^2+b^2)^{1/2}\right)^2$

$= \underline{\underline{\frac{1}{4}M(a^2+b^2)}}$

$I = \frac{1}{4}M(a^2+b^2) + \underbrace{\frac{1}{12}M(a^2+b^2)}$

$= \underline{\underline{\frac{1}{3}M(a^2+b^2)}}$

This has been proven elsewhere, and you should be able to prove w/o any difficulty.

\# 55

$M = 500\,g = 0.5\,kg$

$m = 460\,g = 0.46\,kg$

$R = 5\,cm = 0.05\,m$

$d = 75\,cm$, $t = 5.00\,sec$

(a) $a = -\text{const.}$ —— ①

$V = \int a \cdot dt = -at + \cancel{V_0}^{0}$ —— ②

$y = \int V \cdot dt = -\frac{1}{2}at^2 + \cancel{y_0}^{0.75}$ —— ③

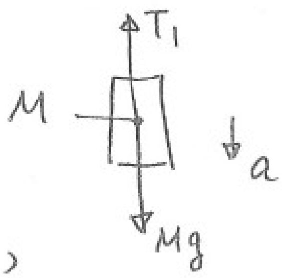
Eqn. ③ $t = 5$, $y = 0$, solve for a.

$0 = -\frac{1}{2}at^2 + 0.75$

$\frac{1}{2}at^2 = 0.75$

$a = \frac{0.75 \cdot 2}{5^2} = \underline{\underline{0.06\ m/sec^2}}$ (downward for M)

(b)

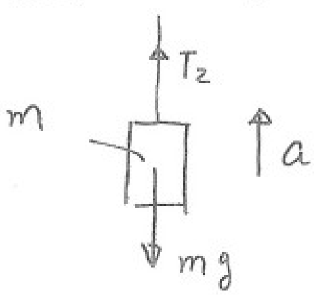


$-Mg + T_1 = -Ma$

$T_1 = Mg - Ma$

$= M(9.81 - 0.06) = \underline{\underline{4.875N}}$

(c)



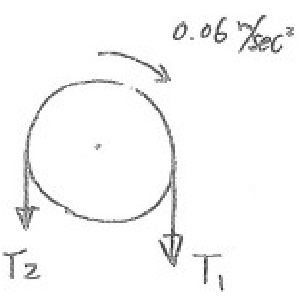
$T_2 - mg = ma$

$T_2 = mg + ma$

$= m(g + a) = 0.46(9.81 + 0.06) = \underline{\underline{4.5402N}}$

(d) $a = r\alpha \Rightarrow \alpha = \frac{a}{r} = \frac{0.06\ m/sec^2}{0.05} = \underline{\underline{1.2\ rad/sec^2\ \text{clockwise}}}$

(e)



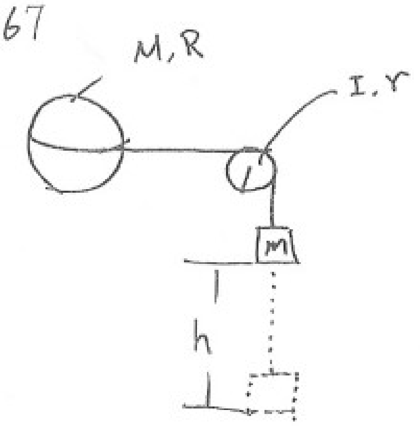
$T_1 \rightarrow$ clockwise
$T_2 \rightarrow$ counterclockwise } they oppose each other

$$\tau = r \times F_{net} = I\alpha$$

$$I = \frac{r \times F_{net}}{\alpha} = \frac{0.05(T_1 - T_2)}{\alpha}$$

$$= \underline{0.01395 \text{ kg m}^2}$$

#67



lost of PE of m is transfered to

KE of m, RE of pulley & RE of the shell.

$$I_{shell} = \frac{2}{3} MR^2$$

$$V = r\omega \Rightarrow \omega = \frac{V}{r}$$

$$\Delta PE = KE + RE_{pully} + RE_{shell}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} I_p \omega_p^2 + \frac{1}{2} I_{shell} \omega_{shell}^2$$

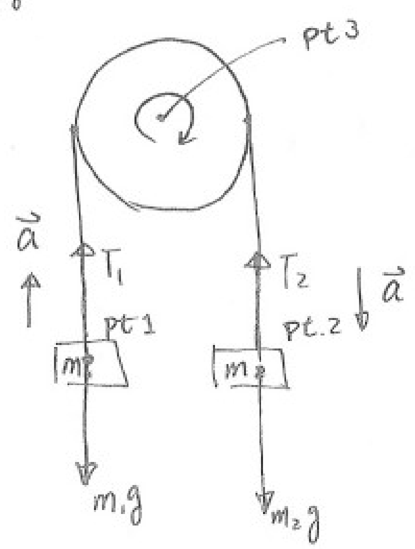
$$= \frac{1}{2}mV^2 + \frac{1}{2} I_p \frac{V^2}{r^2} + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\cdot\left(\frac{V^2}{R^2}\right)$$

$$= \left(\frac{1}{2}m + \frac{1}{2}\frac{I_p}{r^2} + \frac{1}{3}M\right)V^2$$

$$V = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I_p}{r^2} + \frac{1}{3}M}} = \sqrt{\frac{mgh}{\frac{1}{6r^2}(3mr^2 + 3I_p + 12Mr^2)}}$$

$$= \sqrt{\frac{6r^2 mgh}{3mr^2 + 3I_p + 12Mr^2}}$$

#78



Since $m_1 < m_2$, the pulley will rotate clockwise.

pt 1

$$T_1 - m_1 g = m_1 a \Rightarrow T_1 = m_1 a + m_1 g \quad \text{—— ①}$$

pt 2

$$T_2 - m_2 g = -m_2 a \rightarrow T_2 = m_2 g - m_2 a \quad \text{—— ②}$$

pt 3

$$\tau = \vec{r} \times \vec{F} = I \cdot \alpha \quad \text{—— ③}$$

$$\underset{\text{(clockwise)}}{RT_2} - \underset{\text{(counter clockwise)}}{RT_1} = I\alpha$$

$$R(m_2 g - m_2 a) - R(m_1 a + m_1 g) = I_{Disk}\alpha \qquad \left(\begin{array}{l} I_{Disk} = \frac{1}{2} m_{Disk} R^2 \\ \alpha = \frac{a}{R} \end{array} \right)$$

$$R(m_2 g - m_2 a) - R(m_1 a + m_1 g) = \frac{1}{2} m_D R^2 \frac{a}{R}$$

$$m_2 g - m_2 a - m_1 a - m_1 g = \frac{1}{2} m_D a$$

$$m_2 g - m_1 g = m_2 a + m_1 a + \frac{1}{2} m_D a$$

(a)

$$a = \frac{(m_2 - m_1) g}{(m_2 + m_1 + \frac{1}{2} m_D)} = \underline{\underline{1.5696 \ m/sec^2}} \ \text{——} \ ③$$

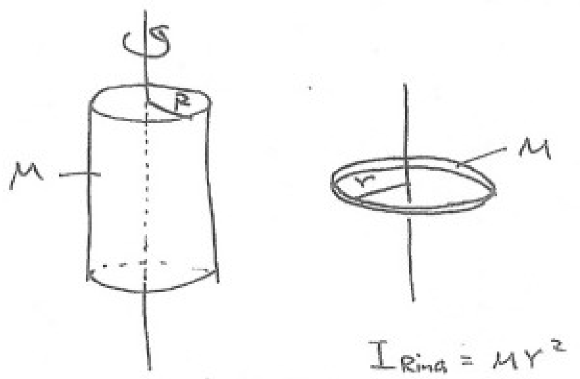
(b) ① ← ③

$$T_1 = m_1 a + m_1 g = \underline{\underline{4.55184 N}}$$

(C) ② ← ③

$$T_2 = m_2 g - m_2 a = \underline{\underline{4.94424 N}}$$

#85



$I_{Ring} = Mr^2$

$I_{cyl.} = \frac{1}{2} MR^2$

(a) If

$$I_{cyl.} = I_{Ring}$$

$$\frac{1}{2} MR^2 = Mr^2$$

$$r^2 = \frac{1}{2} R^2$$

$$\underline{\underline{r = \frac{R}{\sqrt{2}}}}$$

(b)

$$K = \sqrt{\frac{I}{M}}$$

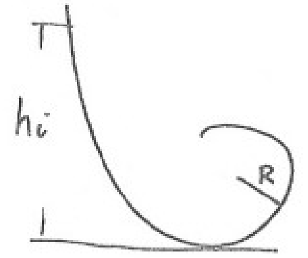
$$(K)^2 = \left(\sqrt{\frac{I}{M}}\right)^2$$

$$K^2 = \frac{I}{M}$$

$$\underline{\underline{\therefore I = MK^2}}$$

ch 11 # 8, 39, 47, 51, 56, 64, 100

8



Go back to Page 161 #6 and identify similarities & differences

(a) On the top of the loop, its own weight is causing the centripetal force ($\Rightarrow N = 0$)

$$F_{centri} = mg = m\frac{V^2}{R}$$

$$V^2 = gR \quad \text{———} \quad ①$$

Cons. of E

E_i $\qquad$ E_f

$$PE_i = PE_f + KE + RE$$

$$mgh_i = mgh_f + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

$$\left[\begin{array}{l} I_{\text{solid sphere}} = \frac{2}{5}mr^2 \\ V = r\omega \Rightarrow \omega = \frac{V}{r} \end{array}\right]$$

$$mgh_i = mg(2R) + \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{V^2}{r^2}$$

This is one of the reasons why I've been stressing about upper & lower cases.

$$gh_i = 2gR + \frac{1}{2}V^2 + \frac{1}{5}V^2$$

$$gh_i = 2gR + \frac{7}{10}V^2 \quad \text{——} \quad ②$$

Remember seeing this in the lab?

② ← ①

$$gh_i = 2gR + \frac{7}{10}(gR)$$

$$\underline{\underline{h_i = \frac{27}{10}R}}$$

(b) E_i

$$PE_i = PE_f + KE + RE$$

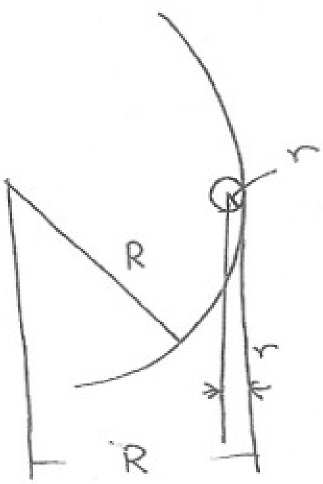
$$mg(6R) = mgR + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

$$6mgR = mgR + \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\cdot\frac{V^2}{r^2}$$

$$5mgR = \frac{1}{2}mV^2 + \frac{1}{5}mV^2$$

$$5gR = \frac{7}{10}V^2$$

$$V^2 = \frac{50}{7}gR \quad \text{———} \quad (1)$$



$$F_{centri} = m\frac{V^2}{d}$$

d : dist. between the Center of the Loop to the Center of the ball

$$= m\frac{V^2}{R-r} \quad \text{——} \quad (2)$$

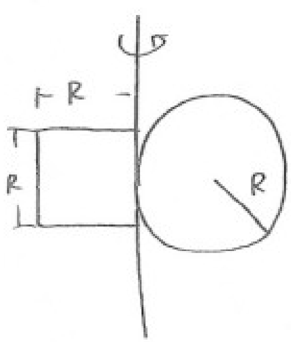
$$(2) \leftarrow (1)$$

$$= m\frac{\frac{50}{7}gR}{R-r}$$

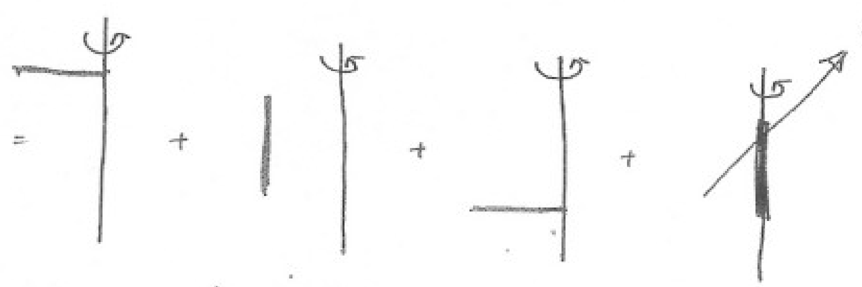
For $R >> r$

$$F = m\frac{\frac{50}{7}g\cancel{R}}{\cancel{(R-r)}} = \underline{\underline{\frac{50}{7}gm}}$$

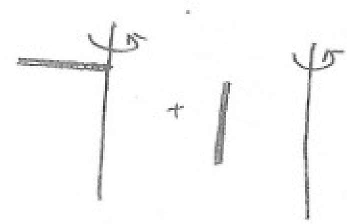
#39



I_{square} =

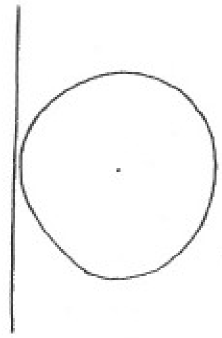


= 2 ×



$$= 2 \times \frac{1}{3}mR^2 + mR^2$$

$$= \frac{2}{3}mR^2 + mR^2 = \underline{\frac{5}{3}mR^2}$$



I_{hoop}

$= I_{cm} + I_{\text{hoop WRT its own center}}$

$I_{\text{hoop w.R.T. its center}}$ is derived elsewhere

$= \frac{1}{2} m R^2$

$= mR^2 + \frac{1}{2} mR^2$

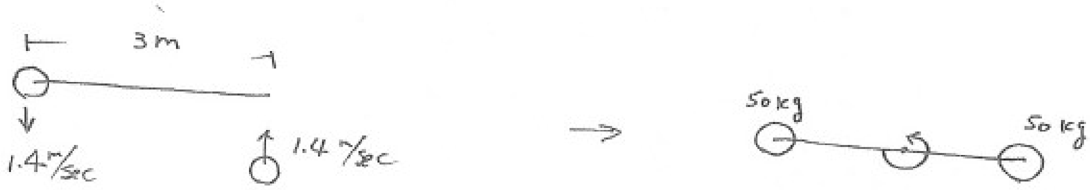
$= \underline{\underline{\frac{3}{2} m R^2}}$

$I_{Total} = I_{square} + I_{hoop}$

$= \frac{5}{3} mR^2 + \frac{3}{2} m R^2$

$= \frac{19}{6} m R^2 = \frac{19}{6} \, 2 \cdot (0.5)^2 = \underline{\underline{1.58\dot{3} \text{ kg m}^2}}$

#47



(a) they start rotating w.R.T. the center of rod, 1.5 m away from them.
At it was shown in #27, $\vec{l} = mvd$

$l_i = l_f$

$mvd = I\omega$ $\quad \left(I = 2 \times mr^2 = 2 \times m \left(\frac{1}{2} d\right)^2 = \frac{1}{2} m d^2 \right)$

$\omega = \frac{mvd}{I}$

$= \frac{\cancel{m} v \cancel{d}}{\frac{1}{2} \cancel{m} d^{\cancel{2}}} = \frac{1.4}{\frac{1}{2} \cdot 3} = \underline{\underline{0.9\dot{3} \text{ rad/sec}}}$

(b)

$KE = 2 \cdot \frac{1}{2} m v^2 = 50 \cdot (1.4)^2 = \underline{\underline{98 J}}$

$RE = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} m d^2\right) \cdot (0.9\dot{3})^2 = \underline{\underline{98 J}}$ $\Big] \rightarrow$ Energy Conserves!

(C) Cons. of ℓ

$$I_i \omega_i = I_f \omega_f$$

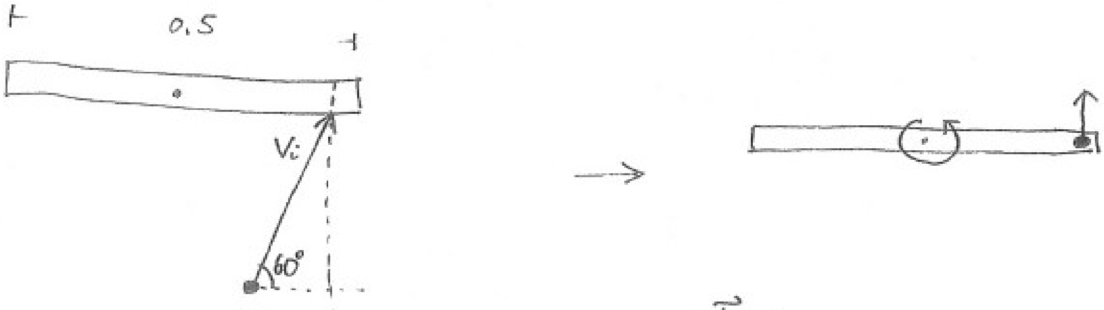
$$\omega_f = \frac{I_i \omega_i}{I_f} = \underline{\underline{8.4 \text{ rad/sec}}}$$

$$\left[\begin{array}{ll} I_i = \frac{1}{2} m d_i^2 & d_i = 3m \\ I_f = \frac{1}{2} m d_f^2 & d_f = 1m \\ \omega_i = 0.9\dot{3} \text{ rad/sec} & \end{array}\right]$$

(d) $RE = \frac{1}{2} I_f \omega_f^2 = \underline{\underline{881.9 \text{ J}}}$

(e) the diff. comes from skaters. they did work by <u>pulling</u> themselves toward center.

#51



$$\vec{\ell}_i = \vec{r} \times \vec{p} = 0.25 \cdot \overbrace{m_b V_i \sin 60°}^{\vec{p}_{\perp \vec{r}}}$$

$$\vec{\ell}_f = I_{rod}\, \omega_f + I_{bullet}\, \omega_f$$

$$= \frac{1}{12} M_R L_R^2 \cdot \omega_f + m_b \left(\tfrac{1}{2} L_R\right)^2 \cdot \omega_f$$

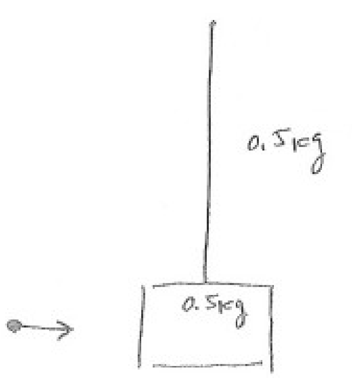
$$\vec{\ell}_i = \vec{\ell}_f$$

$$0.25\, m_b V_i \sin 60° = \frac{1}{12}(4)(0.5)^2 \cdot 10 + 0.003\left(\tfrac{1}{2}\, 0.5\right)^2 \cdot 10$$

$$V_i = \frac{\frac{1}{12}(4)(0.5)^2 \cdot 10 + 0.003\left(\frac{1}{2}\, 0.5\right)^2 \cdot 10}{0.25 \cdot 0.003 \sin 60°}$$

$$= \underline{\underline{1.29 \times 10^3 \text{ m/sec} \quad (1.294547604 \times 10^3 \text{ m/sec})}}$$

#56



$I_{rod} = 0.06$ kg m^2

(a) $I_T = I_{Rod} + I_{block} + I_{bullet}$

$= 0.06\ kg\,m^2 + m_{block} r^2 + m_{bullet} \cdot r^2$

$= [0.06 + 0.5(0.6)^2 + (0.001)(0.6)^2]\ kg\,m^2$

$= \underline{\underline{0.24036\ kg\,m^2}}$

(b) Cons. of $\vec{l}$

$\vec{l}_i = \vec{l}_f$

$\vec{r} \times m_{bullet} V = I\,\omega_f$

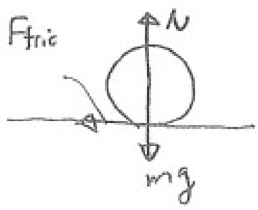
$V = \frac{I_T \omega_f}{r \cdot m_{bullet}} = \frac{0.24036 \cdot 4.5}{0.6 \cdot 0.001} = \underline{\underline{1.8027 \times 10^3\ m/sec}}$

#64

(a) $V_{com,f} = R\omega_f$

$= \underline{\underline{(0.11)\,\omega_f}}$ (clockwise)

(b) linear acc. is due to kinetic friction.



x	y
$-F_{fric} = -ma$	$N - mg = 0$
$-\mu_k N = -ma$	$N = mg$

$a = \frac{\mu_k m g}{m} = \mu_k g = \underline{\underline{2.0601\ m/sec^2}}$ (to negative x direction)

(c) the Kinetic friction is causing the ball to rotate

$\Rightarrow$ Kinetic friction (Force) causes "Torque".

$\tau = I\alpha = R F_{fric}$

$\alpha = \frac{R F_{fric}}{I} = \frac{\cancel{R} \mu_k \cancel{m} g}{\frac{2}{5} \cancel{M} R^{\cancel{2}}} = \frac{(0.21)(9.81)}{\frac{2}{5}(0.11)} = \underline{\underline{46.82045455\ rad/sec^2}}$

(clockwise)

(d) there are two ways to solve

(I) While the ball is slowing down linearly (question (b)), it gains its angular speed (question (c)). Since we know how to convert from linear to angular (or vice versa), we set their speeds to be the same.

Linear

$a = -2.0601 \ m/sec^2$ ——— (1)

$V = \int a \cdot dt = -2.0601t + \cancel{V_0}^{8.5}$ ——— (2)

$x = \int V \cdot dt = -2.0601 \cdot \frac{1}{2}t^2 + 8.5t + \cancel{x_0}^{0}$ ——— (3)

Angular (see (c))

$\alpha = \frac{R F_{fric}}{I} = \frac{\mu_k g}{\frac{2}{5}R}$ ——— (4)

$\omega = \int \alpha \cdot dt = \frac{\mu_k g}{\frac{2}{5}R} t + \cancel{\omega_0}^{0}$ ——— (5)

$\theta = \int \frac{\mu_k g}{\frac{2}{5}R} t \cdot dt = \frac{1}{2}t^2 \frac{\mu_k g}{\frac{2}{5}R}$ ——— (6)

Eqn (2) change it to ω

$$V = \omega R \Rightarrow \omega = \frac{V}{R} = \frac{-2.0601t + 8.5}{R} \quad \text{——— (2)'}$$

their final ∡ speed should be the same

$\therefore$ (2)' = (5)

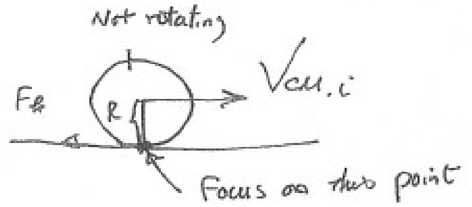
$$\frac{-2.0601t + 8.5}{R} = \frac{\mu_k g}{\frac{2}{5}R} t$$

$$\frac{8.5}{R} = \frac{\mu_k g t}{\frac{2}{5}R} + \frac{2.0601}{R} t = \left(\frac{\mu_k g}{\frac{2}{5}R} + \frac{2.0601}{R}\right) t$$

$$\therefore t = \frac{8.5}{R\left(\frac{\mu_k g}{\frac{2}{5}R} + \frac{2.0601}{R}\right)} = \frac{8.5}{0.11\left(\frac{0.21 \cdot 9.81}{\frac{2}{5} \cdot 0.11} + \frac{2.0601}{0.11}\right)}$$

$$= \underline{\underline{1.17880943 \ sec}} \quad \text{——— (2)''}$$

(II) The second way (and the way I like better personally) is to use the "Conservation of $\vec{l}$":



l_i ← Due to its linear motion

l_f ← due to its linear motion

← due to its angular motion

$$I\omega_{(\text{of the ball})} = m V_{cm,f} R + I\omega_f$$

$$m V_{cm,i} \cdot R = m V_{cm,f} R + \frac{2}{5} m R^2 \cdot \omega_f \leftarrow \omega_f = \frac{V_{cm,f}}{R}$$

$$V_{cm,i} = V_{cm,f} + \frac{2}{5} V_{cm,f}$$

$$V_{cm,i} = \frac{7}{5} V_{cm,f} \Rightarrow V_{cm,f} = \frac{5}{7} V_{cm,i}$$

$$\boxed{\omega_f = \frac{V_{cm,f}}{R} = \frac{\frac{5}{7} V_{cm,i}}{R}}$$

Also, $\omega_f = \alpha t = \dfrac{\mu_k g}{\frac{2}{5} R} t$ (see (c))

$$\therefore \omega_f = \frac{\frac{5}{7} V_{cm,i}}{R} = \frac{\mu_k g}{\frac{2}{5} R} t$$

$$t = \frac{\frac{5}{7} V_{cm,i} \cdot \frac{2}{5} R}{R \cdot \mu_k g} = \frac{\frac{2}{7} V_{cm,i}}{\mu_k g}$$

$$= \frac{\frac{2}{7} \cdot 8.5}{0.21 \cdot 9.81} = \underline{\underline{1.178860943 \text{ sec}}}$$

(e) ③ ← ②''

$$x = -2.0601 \cdot \frac{1}{2} t^2 + 8.5t$$

$$= \underline{\underline{8.588531253 \text{ m}}}$$

(f) ② ← ②''

$$V = -2.0601 t + 8.5 = \underline{\underline{6.071534693 \text{ m/sec}}}$$

100

$$\vec{\tau} = \vec{R} \times \vec{F} = I \cdot \alpha$$

$$\therefore \int \tau \cdot dt$$

$$\int \vec{r} \times \vec{F} \cdot dt$$

$$= rFt \quad \text{(both } r \text{ \& } F \text{ are const.)}$$

$$\int I \cdot \alpha \cdot dt$$

$$= I \int \alpha \cdot dt$$

$$= I \; \omega \Big|_{\omega_i \; (t=i)}^{\omega_f \; (t=f)}$$

$$= I(\omega_f - \omega_i)$$

Additional Problems Ch. 11.

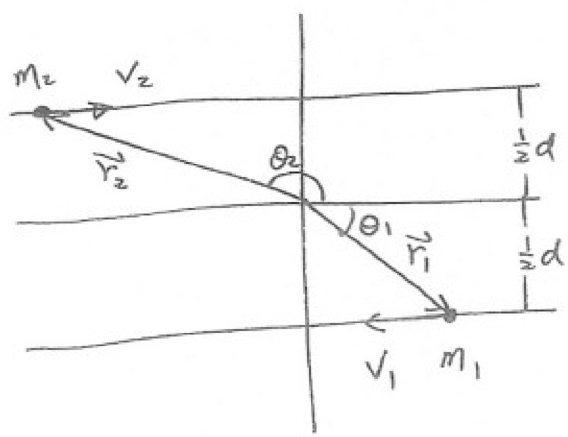
#1 Two particles, each of mass m and speed v travel in opposite directions along parallel lines separated by a distance d. (a) In terms of m, v, and d, find an expression for the magnitude L of the angular momentum of the two-particle system around a point midway between the two lines. (b) Does the expression change if the point about which L is calculated is not midway between the lines? (c) Now reverse the direction of travel for one of the particles and repeat (a) and (b).

#2 Two cylinders having radii R_1 and R_2 and rotational inertias I_1 and I_2 about their central axes are supported by axles perpendicular to the plane. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually, slipping ceases, and the two cylinders rotate at constant rates in opposite directions. Find the final angular velocity ω_2 of the small cylinder in terms of I_1, I_2, R_1, R_2, and ω_0.

#3 A wheel is rotating freely at angular speed 800 rev/min of a shaft whose rotational inertial is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first, is suddenly coupled to tht same shaft. (a) What is the angular speed of the resultant combination of the shaft and two wheels? (b)What fraction of the original rotational kinetic energy is lost?

#1

(a)



Note:
I am using abs. values for α's and taking care of signs by realizing the directions of angular momenta

$$\vec{l}_T = \vec{l}_1 + \vec{l}_2$$

$$= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$= m_1(\vec{r}_1 \times \vec{v}_1) + m_2(\vec{r}_2 \times \vec{v}_2)$$

[I like clockwise motion positive. The book shows clockwise negative]

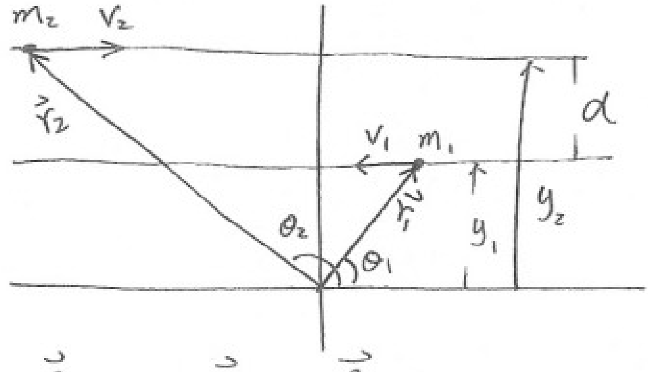
$$= m_1(r_1 v_1 \sin\theta_1) + m_2(r_2 v_2 \sin\theta_2) \qquad r_1 \sin\theta_1 = \tfrac{1}{2}d$$

$$= m v(\tfrac{1}{2}d) + m v(\tfrac{1}{2}d) \qquad r_2 \sin\theta_2 = \tfrac{1}{2}d$$

$$m_1 = m_2$$

$$= \underline{\underline{m v d}} \qquad |v_1| = |v_2|$$

(b)



$$\vec{\ell}_T = \vec{\ell}_1 + \vec{\ell}_2$$

$$= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$= -m_1(\vec{r}_1 \times \vec{v}_1) + m_2(\vec{r}_2 \times \vec{v}_2) \qquad m_1 = m_2$$

counterclockwise   clockwise

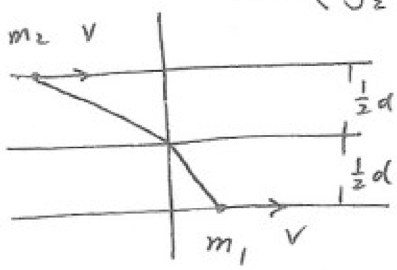
$$= -m(r_1 v_1 \sin\theta_1) + m(r_2 v_2 \sin\theta_2) \qquad r_1 \sin\theta_1 = y_1, \quad r_2 \sin\theta_2 = y_2$$

$$= -m v y_1 + m v y_2 \qquad |v_1| = |v_2|$$

$$= m v(y_2 - y_1) = \underline{m v d}$$

<u>The same result as (a)</u>

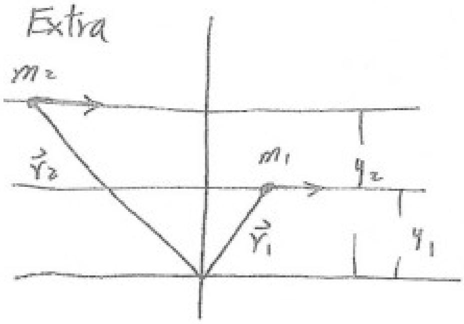
(C)



The set up eqn. is same as (a) except for a sign of $\vec{\ell}_1$ because it is rotating counterclockwise W.R.T. the origin

$$\vec{\ell}_T = -m v \tfrac{1}{2}d + m v \tfrac{1}{2}d = \underline{\underline{0}}$$

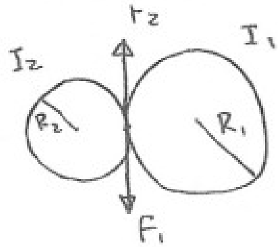
(d) Extra



$$\vec{\ell} = m v y_1 + m v y_2$$

$$= \underline{m v(y_1 + y_2)}$$

#2



F_1 is a frictional force working on a larger wheel (so that it will slow down)

F_2 is a frictional force working on a smaller wheel (so that it will speed up)

F_1 & F_2 are equal and opposite and they exist until the wheels' linear vels. at contact pt. are equal (& opposite)
(angular speeds are <u>not</u> equal & opposite)

$$\tau_1 = \vec{R}_1 \times \vec{F}_1 \qquad \tau_2 = \vec{R}_2 \times \vec{F}_2$$

$$\Delta l = \int \tau_1 \, dt \qquad \Delta l = \int \tau_2 \, dt$$

counterclockwise

$$= R_1 \cdot (-F) t \qquad = R_2 F t$$

$$I_1(\omega_{1f} - \omega_{1i}) = -R_1 F t \quad — ①$$

$$I_2(\omega_{2f} - \cancel{\omega_{2i}}^{\,0}) = R_2 F t$$

$$t = \frac{I_2 \omega_{2f}}{R_2 F} \quad — ③$$

① ← ②

$$I_1 (\omega_{1f} - \omega_{1i}) = - R_1 F \frac{I_2 \omega_{2f}}{R_2 F}$$

$$I_1 (\omega_{1f} - \omega_{1i}) = - \frac{R_1}{R_2} I_2 \omega_{2f}$$

Also, at the contact pt., their linear speeds should be the same:

$$V = R_1 \omega_{1f} = R_2 \omega_{2f}$$

$$\therefore \omega_{1f} = \frac{R_2}{R_1} \omega_{2f}$$

$$I_1 \left(\frac{R_2}{R_1} \omega_{2f} - \omega_{1i} \right) = - \frac{R_1}{R_2} I_2 \omega_{2f}$$

$$\frac{R_2}{R_1} I_1 \omega_{2f} - I_1 \omega_{1i} = - \frac{R_1}{R_2} I_2 \omega_{2f}$$

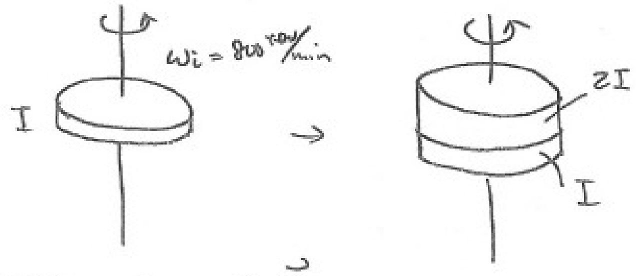
$$\frac{R_2}{R_1} I_1 \omega_{2f} + \frac{R_1}{R_2} I_2 \omega_{2f} = I_1 \omega_{1i}$$

$$\omega_{2f} \left(\frac{R_2}{R_1} I_1 + \frac{R_1}{R_2} I_2 \right) = I_1 \omega_i$$

$$\omega_{2f} \left(\frac{R_2^2 I_1 + R_1^2 I_2}{R_1 R_2} \right) = I_1 \omega_i$$

$$\omega_{2f} = \left(\frac{R_1 R_2}{R_2^2 I_1 + R_1^2 I_2} \right) I_1 \omega_i$$

#3



(a) Cons of $\vec{\ell}$

$$\vec{\ell}_i = \vec{\ell}_f$$

$$I_1 \omega_i = I_1 \omega_f + I_2 \omega_f \qquad (I_2 = 2 I_1)$$

$$\therefore I_1 \omega_i = (I_1 + 2I_1)\,\omega_f$$

$$\omega_f = \frac{I_1 \omega_i}{3 I_1} = \frac{1}{3}\, 800 \text{ rev/min} = \underline{\underline{2.\dot{6} \times 10^2 \text{ rev/min}}}$$

$$= \underline{\underline{27.925268 \text{ rad/sec}}}$$

(b) $KE_i = \frac{1}{2} I_1 \omega_i^2$

$$KE_f = \frac{1}{2} I_1 \omega_f^2 + \frac{1}{2} I_2 \omega_f^2$$

$$= \frac{1}{2} I_1 \omega_f^2 + \frac{1}{2}(2I_1)\,\omega_f^2$$

$$= \frac{1}{2}(3 I_1)\,\omega_f^2 \qquad (\omega_f = \frac{1}{3}\omega_i^2 \text{ from (a)})$$

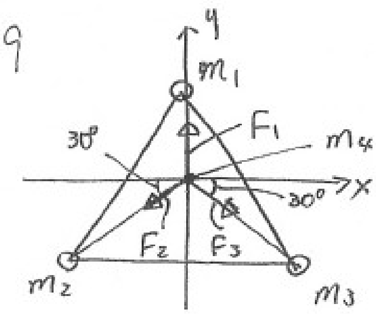
$$= \frac{1}{2}(3I_1)\left(\frac{1}{3}\omega_i\right)^2$$

$$= \frac{1}{6} I_i \omega_i^2$$

$$\frac{\Delta KE}{KE_i} = \frac{\frac{1}{2} I_1 \omega_i^2 - \frac{1}{6} I_i \omega_i^2}{\frac{1}{2} I \omega_i^2} = \underline{\underline{\frac{2}{3}}}$$

(It lost $\frac{2}{3}$ of the original E) ch 13 # 9, 32, 42, 103

#9



This setting is exactly the same as the Lab #1, first set up (3 forces separated by 120° from each)
So, $\Sigma \vec{F} = 0$, and $|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3|$ is what we expect to see.

$$F_1 = G\frac{m_1 m_4}{r^2}\left(0\hat{i} + 1\hat{j}\right)$$

$$F_2 = G\frac{m_2 m_4}{r^2}\left(-\cos 30^\circ\hat{i} - \sin 30^\circ\hat{j}\right)$$

$$F_3 = G\frac{m_3 m_4}{r^2}\left(\cos 30^\circ\hat{i} - \sin 30^\circ\hat{j}\right)$$

$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

(a) x comp

$$F_{1x} + F_{2x} + F_{3x} = 0 + G\frac{m_2 m_4}{r^2}(-\cos 30^\circ) + G\frac{m_3 m_4}{r^2}(\cos 30^\circ) = \underline{\underline{0}}$$

y comp

$$F_{1y} + F_{2y} + F_{3y} = G\frac{m_1 m_4}{r^2} + G\frac{m_2 m_4}{r^2}(-\sin 30^\circ) + G\frac{m_3 m_4}{r^2}(-\sin 30^\circ) = 0$$

$$m_2 = m_3 \equiv m$$

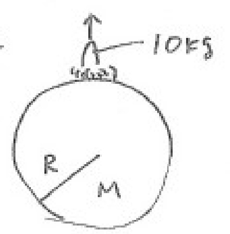
$$= G\frac{m_1 m_4}{r^2} - 2\frac{m m_4}{r^2}\sin 30^\circ$$

$$\therefore G\frac{m_1 m_4}{r^2} = 2\frac{m m_4}{r^2}\cancel{\sin 30^\circ}\ \tfrac{1}{2}$$

$$\underline{\underline{m_1 = m}}$$

(b) magnitude of each force will be double, but F_{net} is still "zero".

#32



$M = 5\times10^{23}$ kg
$R = 3\times10^{6}$ m

E_i E_f

$$KE_i = KE_f + PE$$

$$\therefore KE_f = KE_i - PE$$

(a)

$$KE_f = KE_i - \int_{3\times10^6}^{4\times10^6} G\frac{m_1 m_2}{r^2}\cdot dr$$

(Don't ever think that PE is always equal to mgh !)

$$= 5\times10^7 J - 2.78025\times10^7 J$$

$$= \underline{\underline{2.21975\times10^7 J}}$$

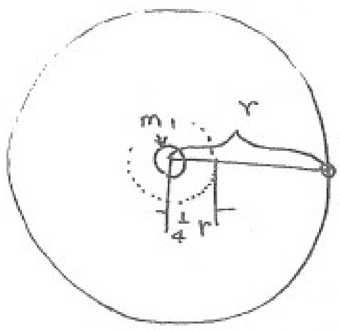
(b)

$$\overset{E_i}{KE} = \overset{E_f}{PE}$$

$$PE = \int_{3\times10^6}^{8\times10^6} G\frac{m_1 m_2}{r^2}dr = Gm_1m_2\left(\frac{1}{3\times10^6} - \frac{1}{8\times10^6}\right)$$

$$= \underline{\underline{6.950625\times10^7 J}}$$

#42



$$F = G\frac{m_1m_2}{r^2} = m_2\frac{V^2}{r}$$

$$\therefore V = \sqrt{\frac{Gm_1}{r}}$$

$$P = \frac{2\pi r}{V} = \frac{2\pi r}{\sqrt{\frac{Gm_1}{r}}} = \sqrt{\frac{4\pi^2 r^3}{Gm_1}} = \sqrt{\frac{4\pi^2}{Gm_1}}\cdot r^{3/2}$$

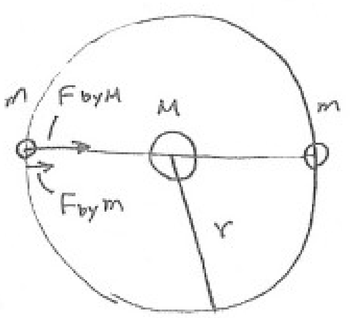
$\Rightarrow$ Orbital Period $\propto r^{3/2}$

Now radius $= \frac{1}{2}r$

$$P \propto \left(\frac{1}{2}r\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} r^{3/2}$$

$$= \frac{1}{\sqrt{8}} r^{3/2} \Rightarrow \underline{\underline{\frac{1}{\sqrt{8}}\text{ Lunar month}}}$$

#103



$$F_{net} = F_{by\,M} + F_{by\,m}$$

$$= G\frac{mM}{r^2} + G\frac{mm}{(2r)^2}$$

$$= G\frac{mM}{r^2} + G\frac{m^2}{4r^2}$$

$$= \frac{Gm}{4r^2}(4M+m) = F_{centri} = m\frac{V^2}{r}$$

$$\therefore \frac{G\cancel{m}}{4r^{\cancel{2}}}(4M+m) = \cancel{m}\frac{V^2}{\cancel{r}}$$

$$\therefore V = \sqrt{\frac{G}{4r}(4M+m)}$$

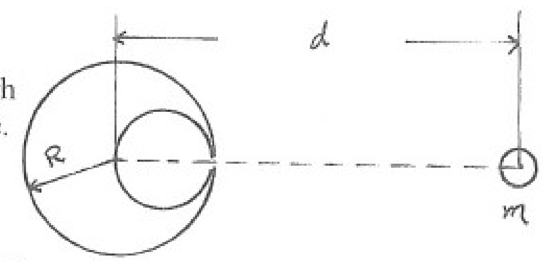
$$P = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{G}{4r}(4M+m)}} = \sqrt{\frac{4\pi^2 r^2}{\frac{G}{4r}(4M+m)}}$$

$$= 4\pi\sqrt{\frac{r^3}{G(4M+m)}}$$

$$= 4\pi\left(\frac{r^3}{G(4M+m)}\right)^{1/2}$$

Ch. 13 additional problems:

#1 The diagram on the right shows a spherical hollow inside a lead sphere of radius R; the surface of the hollow passes through the center of the sphere and "touches" the right side of the sphere. The mass of the sphere before hollowing was M. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass m that lies at a distance d from the center of the lead sphere, on the straight line connecting the centers of the sphere and of the hollow?



#2 One model for a certain planet has a core of radius R and mass M surrounded by an outer shell of inner radius R, outer radius 2R, and mass 4M. If $M - 4.1 \times 10^{24}$ kg and $R = 6.0 \times 10^{6}$ m, what is the gravitational acceleration of a particle at points (a) R and (b) 3R from the center of the planet?

#3 A uniform solid sphere of radius R produces a gravitational acceleration of a_g on its surface. At what two distances from the center of the sphere is the gravitational acceleration $a_g/3$?

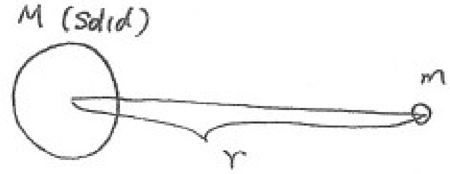
#4 A projectile is fired vertically from Earth's surface with an initial speed of 10 km/sec. Neglecting air drag, how far above the surface of Earth will it go?

#5 Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. At what speed must they move if they all revolve under the influence of one another's gravitational force in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?

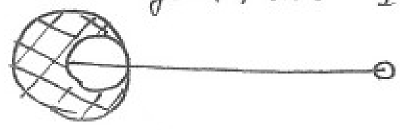
#6 Show that if an object is in an elliptical orbit with semi-major axis about a planet of mass M, then its distance r from the planet and speed v are related by $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$

#7 One way to attack a satellite in Earth orbit is to launch a swarm of pellets in the same orbit as the satellite but in the opposite direction. Suppose a satellite in a circular orbit 500 km above Earth's surface collides with a pellet having mass 4.0 g. (a) What is the kinetic energy of the pellet in the reference frame of the satellite just before the collision? (b) What is the ratio of this kinetic energy to the kinetic energy of a 4.0 g bullet from a modern army rifle with a muzzle speed of 950 m/sec?

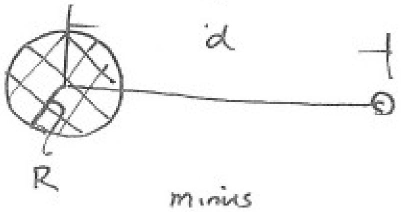
#1



How can we figure out the gravitational force when the object is not a nice shape? One way is to find the cm of the object, but I will use another way.



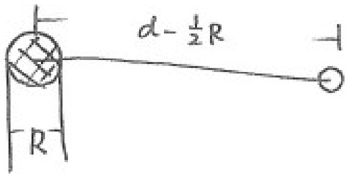
=



$\Rightarrow \quad G\dfrac{Mm}{d^2}$

minus

minus



$\Rightarrow \quad G\dfrac{M'm}{(d-\frac{1}{2}R)^2}$ $\quad M' \equiv$ mass of the punched out (small) sphere

M' ?

$M' = \rho \cdot vol$ where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ & $vol = \frac{4}{3}\pi(\frac{1}{2}R)^3$

$\therefore M' = \dfrac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi\left(\frac{1}{2}R\right)^3$

$= \underline{\frac{1}{8} M}$ $\quad \Rightarrow \quad G\dfrac{\frac{1}{8}Mm}{(d-\frac{1}{2}R)^2}$ (Force by the small sphere)

$$F_{net} = G\frac{Mm}{d^2} - G\frac{Mm}{8(d-\frac{1}{2}R)^2}$$

$$= GMm\left(\frac{1}{d^2} - \frac{1}{8(d-\frac{1}{2}R)^2}\right)$$

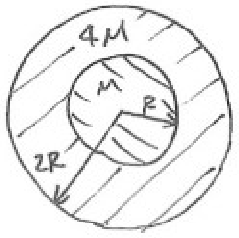
$$= GMm\left(\frac{8(d-\frac{1}{2}R)^2 - d^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

$$= GMm\left(\frac{8d^2 - 8dR + 2R^2 - d^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

$$= GMm\left(\frac{7d^2 - 8dR + 2R^2}{8d^2(d-\frac{1}{2}R)^2}\right)$$

I don't know which one is a simpler form for an answer?

#2



$M = 4.1 \times 10^{24}$ kg

$R = 6 \times 10^{6}$ m

(a) $r = R$

$$a = \frac{G\,M \text{ inside of } r}{r^2}$$

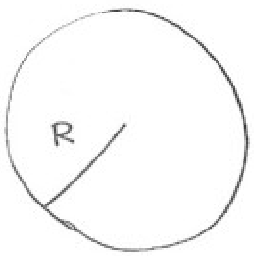
$$= \frac{G\; 4.1\times10^{24}}{(6\times10^{6})^2} = \underline{\underline{7.59935 \text{ m/sec}^2}}$$

(b) $r = 3R$

$$a = \frac{G\,M \text{ inside of } r}{r^2} = \frac{G\,(4M+M)}{(3R)^2} = \frac{5\,GM}{9\,R^2} = \frac{5}{9} \text{ of (a)}$$

$$= \underline{\underline{4.2218611 \text{ m/sec}^2}}$$

#3



Since a is the largest at surface, a decreases both inward & outward.

$$a_{inside} = G\,\frac{M(\text{inside of } r)}{r^2} = G\,\frac{\rho \cdot \text{Vol(inside of } r)}{r^2}$$

$$= G\,\frac{\frac{M}{\frac{4}{3}\pi R^3}\cdot \frac{4}{3}\pi r^3}{r^2}$$

$$= \frac{GM}{R^3}\cdot r$$

$$a = \frac{1}{3}\,\frac{GM}{R^2} = \frac{GM}{R^3}\cdot r$$

$$\underline{\underline{r = \tfrac{1}{3} R}}$$

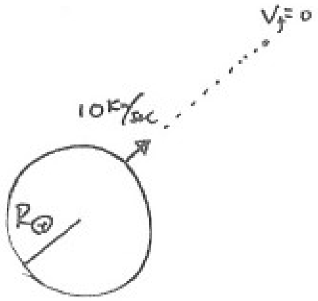
outside

$$a = \frac{1}{3}\,\frac{GM}{R^2} = \frac{GM}{r^2}$$

$$r^2 = 3R^2$$

$$\underline{\underline{r = \sqrt{3}\,R}}$$

4



$R_\oplus = 6.37 \times 10^6 m$

$M_\oplus = 5.98 \times 10^{24} kg$

$V_0 = 1 \times 10^4 m/sec$

$V_f = 0 \, m/sec$

$E_i \quad\quad E_f$

KE = PE

$$\frac{1}{2} m V_i^2 = \int_{R_\oplus}^{r} G \frac{m M_\oplus}{r^2} \cdot dr$$

$$\frac{1}{2} m V_i^2 = G m M_\oplus \left(\frac{1}{R_\oplus} - \frac{1}{r} \right)$$

$$\frac{V_i^2}{2GM_\oplus} = \frac{1}{R_\oplus} - \frac{1}{r}$$

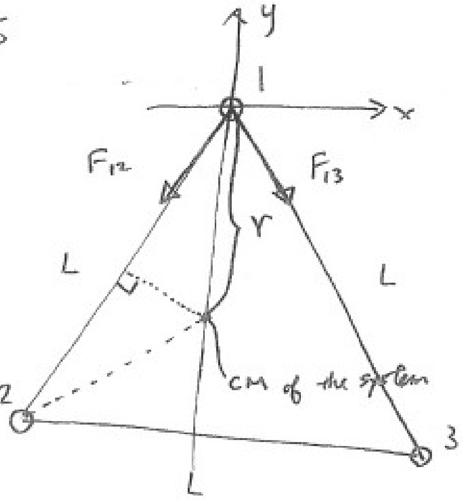
$$\frac{1}{r} = \frac{1}{R_\oplus} - \frac{V_i^2}{2GM_\oplus}$$

$$r = \frac{1}{\frac{1}{R_\oplus} - \frac{V_i^2}{2GM_\oplus}} = 3.156632029 \times 10^7 m$$

So, Above the ground is

$$r - R_\oplus = \underline{2.519632029 \times 10^7 m}$$

5



At star 1

$\Sigma F_x = 0 \quad\quad |F_{12}| = |F_{13}| = G \frac{MM}{L^2}$

$$\Sigma F_y = -F_{12} \cos 30° - F_{13} \cos 30°$$

$$= -2 G \frac{M^2}{L^2} \cos 30°$$

Also,

$$r \cos 30° = \frac{1}{2} L$$

$$r = \frac{L}{2 \cos 30°}$$

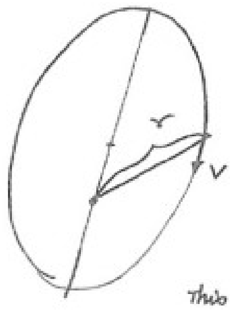
$$\Sigma F_y = F_{centri}$$

$$-2G \frac{M^2}{L^2} \cos 30° = -M \frac{V^2}{r}$$

$$2G \frac{M}{L^2} \cos 30° = \frac{V^2}{\frac{L}{2\cos 30°}} = \frac{V^2 \, 2 \cos 30°}{L}$$

$$\frac{GM}{L} = V^2 \Rightarrow \therefore \underline{V = \sqrt{\frac{GM}{L}}}$$

#6



$$E_{Total} = KE + PE$$

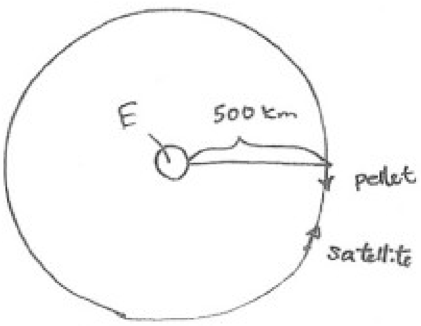
$$-\frac{GMm}{2a} = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

This is proven on P 346.

$$\frac{1}{2}mv^2 = GmM\left(\frac{1}{r} - \frac{1}{2a}\right)$$

$$\therefore V^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

#7



$$F_{grav} = G\frac{M_{\oplus} m}{r^2} = m\frac{v^2}{r}$$

$$V = \sqrt{\frac{GM_{\oplus}}{r}}$$

Both satellite & pellet have the same speed But opposite direction → WRT the satellite, the pellet has twice the speed calculated above.

(a)
$$KE = \frac{1}{2}mV^2$$

$$= \frac{1}{2} m_{pellet} \cdot \left(2\sqrt{\frac{GM_{\oplus}}{r}}\right)^2$$

$$= \frac{1}{2}\ 0.004 \cdot 4 \cdot \frac{6.6726\times10^{-11} \cdot 5.98\times10^{24}}{(6.37\times10^{6} + 500\times10^{3})}$$

$R_{\oplus} + 500km$

$$= 4.646538341\times10^{5} J$$

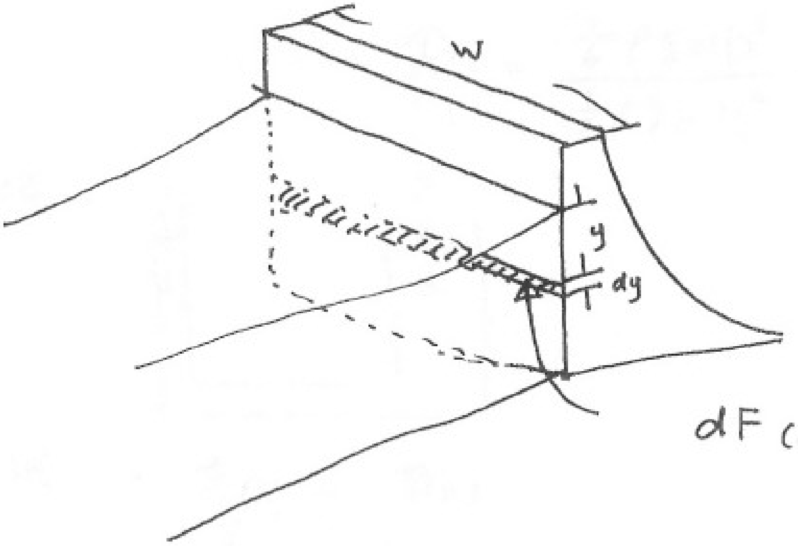
(b)
$$\frac{KE_{in\ space}}{KE_{rifle}} = \frac{4.64\times10^{5} J}{\frac{1}{2} m_{bullet} \cdot V^2}$$

$$= \frac{4.64\cdots\times10^{5} J}{1.805\times10^{3} J}$$

$$= 2.574259468\times10^{2}$$

ch. 14 #19, 22, 31, 37, 59, 61

19



dF (at shaded area) $= P_r \cdot dA$

$= \rho g y \cdot w \cdot dy$

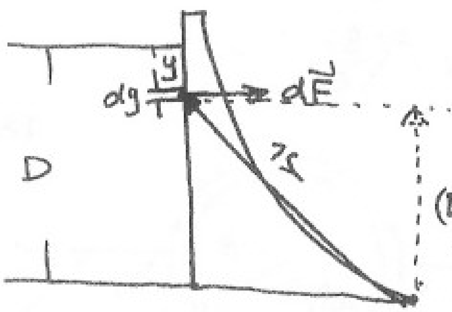
$= \rho g w \cdot y \cdot dy$

(a) $F = \int dF = \int_0^D \rho g w \cdot y \cdot dy$

$= \rho g w \cdot \frac{1}{2} y^2 \Big|_0^D$

$= \underline{\underline{\frac{1}{2} \rho g w D^2}}$

(b)



$d\tau = \vec{r} \times d\vec{F} = (D-y)\, P_r \cdot dA$

$= (D-y)(\rho g y) \cdot w \cdot dy$

This is the perpendicular distance to $d\vec{F}$

$= \rho g w\, (Dy - y^2)\, dy$

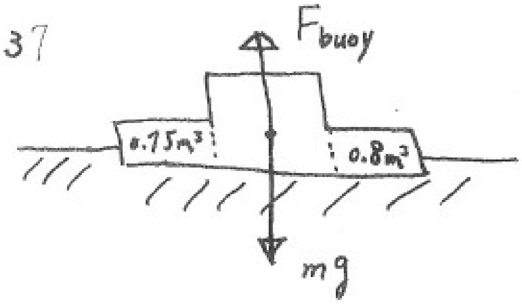
$\therefore \quad \tau = \int d\tau = \int_0^D \rho g w\, (Dy - y^2)\, dy$

$= \rho g w \left(\frac{1}{2} D y^2 - \frac{1}{3} y^3\right)\Big|_0^D$

$= \rho g w \left(\frac{1}{2} D^3 - \frac{1}{3} D^3\right)$

$= \underline{\underline{\frac{1}{6} \rho g w D^3}}$

37



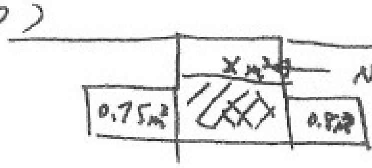
(a)

$$F_{buoy} - mg = 0$$

$$\rho_{H_2O} \cdot V_{H_2O \text{ displaced by the car}} \cdot g - mg = 0$$

$$V_{H_2O} = \frac{m}{\rho_{H_2O}} = \frac{1800 \text{ kg}}{1.0 \times 10^3 \text{ kg/m}^3} = \underline{\underline{1.8 m^3}}$$

(b)

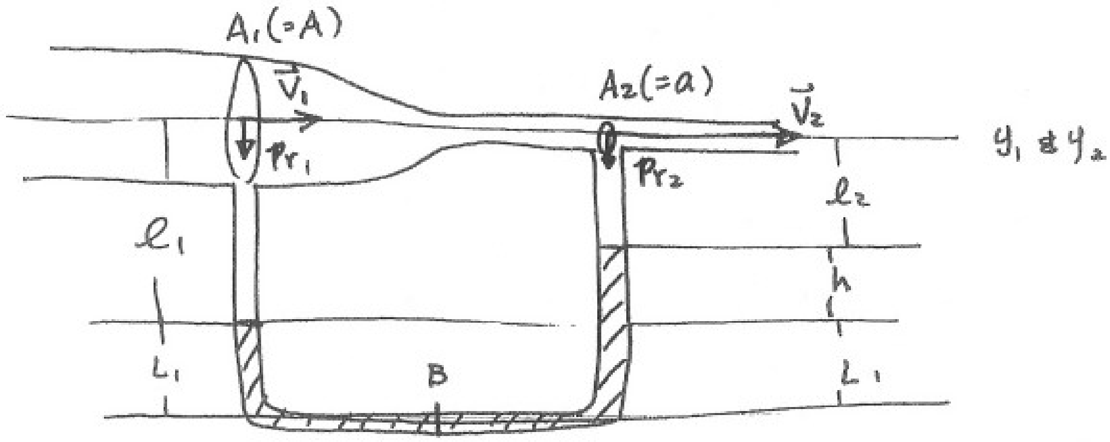


Needed vol. to displace water to be still floating.

$$1.8 m^3 - (0.75 - 0.8) m^3 = 0.25 m^3$$

So, $5 m^3 - 0.25 m^3 = \underline{\underline{4.75 m^3}}$ is the H_2O vol. in the car.

59



$$P_{r_1} + \rho g y_1 + \frac{1}{2} \rho V_1^2 = P_{r_2} + \rho g y_2 + \frac{1}{2} \rho V_2^2 \quad (y_1 = y_2)$$

$$\frac{1}{2} \rho (V_1^2 - V_2^2) = P_{r_2} - P_{r_1}$$

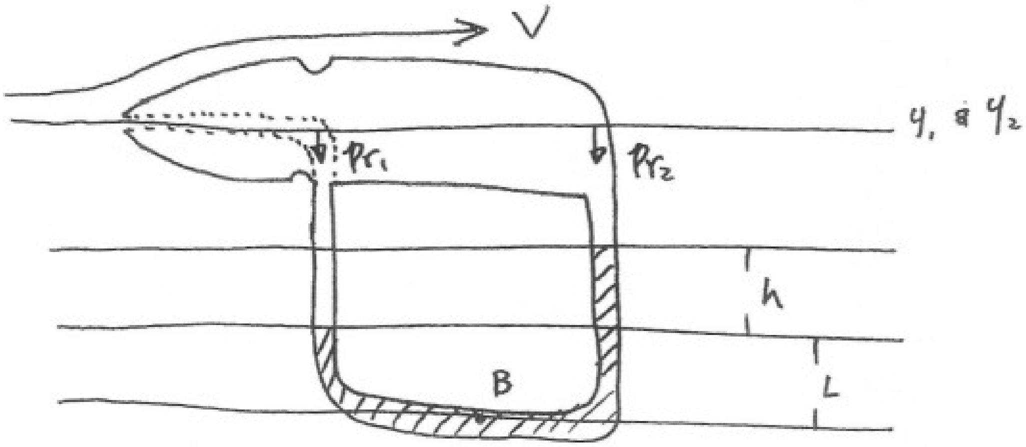
$$\left[\text{Also } A_1 V_1 = A_2 V_2 \quad V_2 = \frac{A_1}{A_2} V_1 \right]$$

$$\frac{1}{2} \rho \left(V_1^2 - \frac{A_1^2}{A_2^2} V_1^2 \right) = P_{r_2} - P_{r_1}$$

$$\frac{1}{2} \rho V_1^2 \left(\frac{A_2^2 - A_1^2}{A_2^2} \right) = P_{r_2} - P_{r_1}$$

$$\therefore V_1 = \sqrt{\frac{2 A_2^2 (P_{r_2} - P_{r_1})}{(A_2^2 - A_1^2) \rho}} \quad \left(= \sqrt{\frac{2 a^2 \Delta P_r}{\rho (a^2 - A^2)}} \right)$$

#61



$$P_{r_1} + \rho_{air} g y_1 + \tfrac{1}{2}\rho_{air} V_1^2 = P_{r_2} + \rho_{air} g y_2 + \tfrac{1}{2}\rho_{air} V_2^2 \quad (y_1 = y_2)$$

$$P_{r_1} + \tfrac{1}{2}\rho_{air} \cancel{V_1^2}^{\,0} = P_{r_2} + \tfrac{1}{2}\rho_{air} V_2^2 \quad \begin{pmatrix} V_1 = 0 \text{ in the tube} \\ V_2 = V \text{ air speed} \end{pmatrix}$$

$$V_2 = \sqrt{\frac{2(P_{r_1} - P_{r_2})}{\rho_{air}}} \quad \text{——— ①}$$

At B.

$$P_r \text{ (Left side)} = P_r \text{ (Right side)}$$

$$P_{r_1} + \rho' g L = P_{r_2} + \rho' g (h + L)$$

$$P_{r_1} - P_{r_2} = \rho' g (h + L) - \rho' g L = \rho' g h \quad \text{— ②}$$

① ← ②

$$V_2 = V_{air\ speed} = \sqrt{\frac{2\rho' g h}{\rho_{air}}}$$

ch 15, # 1, 11, 37, 43, 44, 72, 96, 102

#1

$\nu = 120\ Hz$

$\omega = 2\pi\nu$

(a) 2mm back & forth → Amplitude = $\underline{1mm}$

(b) $x = A \sin(\omega t + \phi)$ — ①

$\dot{x} = A\omega \cos(\omega t + \phi)$ — ②

$\ddot{x} = -A\omega \sin(\omega t + \phi)$ — ③

when $\ddot{x} = 0$, $\dot{x}$ is max

Eqn. ③

$\ddot{x} = -A\omega \sin(\omega t + \phi) = 0$

$\omega t + \phi = n\pi$ (n is an integer) — ③'

② ← ③'

$\dot{x} = A\omega \cos(n\pi)$ 1

$= (0.001)\cdot(2\pi \cdot 120)\cdot 1$

$= \underline{0.7539822369\ m/sec}$

(c) max $\ddot{x}$ is when $\dddot{x} = 0$

$\dddot{x} = -A\omega^3 \cos(\omega t + \phi) = 0$

→ $\cos(\omega t + \phi) = 0$

∴ $(\omega t + \phi) = n\frac{\pi}{2}$ (n is an integer)

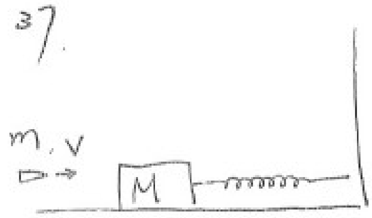
$|\ddot{x}| = |-A\omega^2 \sin(n\frac{\pi}{2})|$

$= (0.001)\cdot(2\pi \cdot 120)^2 \cdot (1)$

$= \underline{5.684892135 \times 10^2\ m/sec^2}$

11. See with the additional problem #3 on this chapter.

37.



(a) cons. of P

p_i p_f

$mv = (m + M)V_f$

$\therefore V_f = \underline{\left(\frac{m}{m+M}\right)v}$

(b) KE upon the impact.

$$\frac{1}{2}(m+M)V_f^2 = \frac{1}{2}(m+M)\left[\left(\frac{m}{m+M}\right)V\right]^2$$

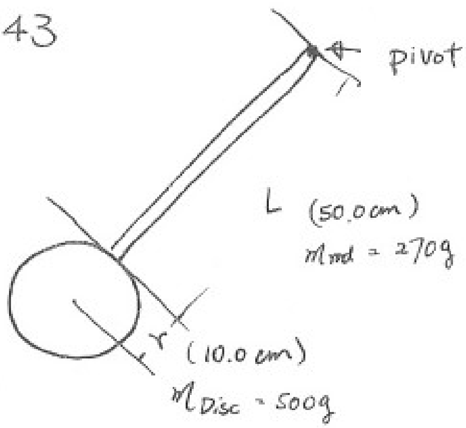
$$= \frac{1}{2}\frac{m^2}{(m+M)}V^2$$

compression of the spring (E conserves)

E_i		E_f
KE.		S.E.
$\frac{1}{2}\frac{m^2}{(m+M)}V^2$	$=$	$\frac{1}{2}kX^2$

$$\therefore X = \sqrt{\frac{m^2V^2}{k(m+M)}} = \underline{\underline{\frac{mV}{\sqrt{k(m+M)}}}}$$

#43



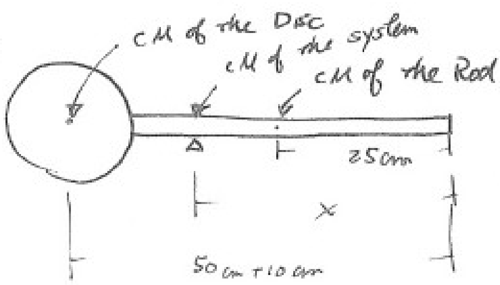
(a) $I = I_{rod} + I_{Disc}$

$$= \frac{1}{3}M_{rod}L_{rod}^2 + \left(\frac{1}{2}M_{Disc}R_{Disc}^2 + M_{Disc}(L_{rod}+R_{Disc})^2\right)$$

$$= \frac{1}{3}(0.270\,kg)(0.5\,m)^2 + \left(\frac{1}{2}(0.5\,kg)(0.1\,m)^2 + (0.5\,kg)(0.5\,m+0.1\,m)^2\right)$$

$$= \underline{\underline{0.205\ kg\,m^2}}$$

(b)



$$X(M_{Disc}+M_{Rod}) = 60\,cm\,(M_{Disc}) + 25\,cm\,(M_{rod})$$

$$X(500g+270g) = 60\,cm\,(500g) + 25\,cm\,(270g)$$

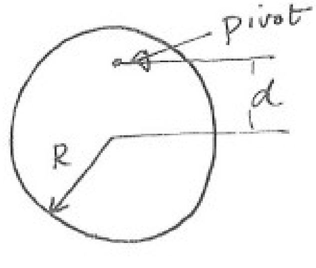
$$\underline{\underline{X = 47.\dot{7}\dot{2}\ cm}}$$

(c) $\omega = \sqrt{\frac{M_T g d}{I}}$ $\quad (d = X)$

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{M_T g d}} = 2\pi\sqrt{\frac{0.205}{(0.27+0.5)\,9.81\cdot 0.47\dot{7}\dot{2}}}$$

$$= \underline{\underline{1.49828\,2212\ sec}}$$

#44



$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$I = I_{CM} + I_{DISC}$$

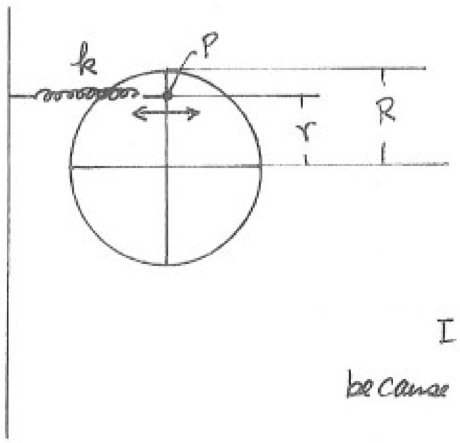
$$= Md^2 + \frac{1}{2}MR^2$$

$$= M\left(d^2 + \frac{1}{2}R^2\right)$$

$$\omega = \sqrt{\frac{Mgd}{I}} = \sqrt{\frac{Mgd}{M(d^2+\frac{1}{2}R^2)}} = \sqrt{\frac{gd}{d^2+\frac{1}{2}R^2}}$$

$$P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^2+\frac{1}{2}R^2}{gd}} = 2\pi\sqrt{\frac{(0.0175)^2 + \frac{1}{2}(0.0235)^2}{9.81 \cdot 0.0175}} = \underline{\underline{0.365955014\ sec}}$$

#72



By the spring force, there is a τ at P

$$\tau = \vec{r} \times \vec{F}$$

$$= \vec{r} \times (-kx) = I\alpha$$

$$I\alpha + rkx = 0$$

because the oscillation is small $\quad x \sim r\theta$

$$\therefore I\alpha + kr^2\theta = 0$$

$$I\ddot{\theta} + kr^2\theta = 0$$

Let $\theta = A\cos(\omega t + \phi)$

$$\dot{\theta} = -A\omega\sin(\omega t + \phi)$$

$$\ddot{\theta} = -A\omega^2\cos(\omega t + \phi)$$

$$\therefore I(-A\omega^2\cos(\omega t+\phi)) + kr^2(A\cos(\omega t+\phi)) = 0$$

$$A(-I\omega^2 + kr^2)\cos(\omega t + \phi) = 0$$

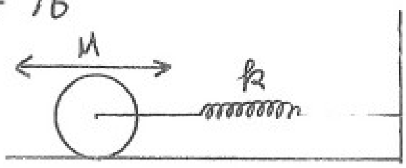
$$\therefore I\omega^2 + kr^2 = 0$$

(a) $\quad \omega = \sqrt{\frac{kr^2}{I}} = \sqrt{\frac{kr^2}{MR^2}} = \frac{r}{R}\sqrt{\frac{k}{M}}$

(b) $\quad r = R \qquad \omega = \sqrt{\frac{k}{M}}$

(c) $\quad r = 0 \qquad \omega = 0$

96



(a) At the equilibrium

$$E_i = E_f$$

$$SE = KE + RE$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 \longrightarrow ①$$

$$I_{cylinder} = \frac{1}{2}MR^2 \longrightarrow ②$$

$$V = \omega R \quad \text{so} \quad \omega = \frac{V}{R} \longrightarrow ③$$

$$① \longleftarrow ② \,\&\, ③$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{4}mV^2$$

when you look at this eqn. you can recognize SE will be change to 2 : 1 ratio of KE : RE.

(a) $KE = \frac{2}{3}SE = \frac{2}{3}\left(\frac{1}{2}kx^2\right) = \frac{2}{3}\left(\frac{1}{2}\cdot 3\cdot(0.25)^2\right) = \underline{\underline{0.0625\,J}}$

(b) $R.E. = \frac{1}{3}SE = \frac{1}{3}\left(\frac{1}{2}kx^2\right) = \underline{\underline{0.03125\,J}}$

(c) $E_{Total} = SE + KE + RE = \text{Constant}$

$$\frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \text{const.}$$

see above

$$\frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{4}mV^2 = \text{const}$$

$$\frac{1}{2}kx^2 + \frac{3}{4}mV^2 = \text{const.}$$

Here is a greate step.

$$\frac{dE_{total}}{dt} = \frac{d\left(\frac{1}{2}kx^2 + \frac{3}{4}mV^2\right)}{dt} = \frac{d\,(\text{const})}{dt} = 0\,!$$

$$\frac{1}{2}\,2kx\cdot\frac{dx}{dt} + \frac{3}{4}\,2mV\cdot\frac{dV}{dt} = 0$$

$$kxV + \frac{3}{2}mV\cdot a = 0$$

$$V\left(kx + \frac{3}{2}ma\right) = 0 \qquad (\text{since } V \text{ is not always zero.})$$

$$kx + \frac{3}{2}ma = 0$$

$$kx + \frac{3}{2}m\ddot{x} = 0$$

Let
$$x = A\cos(\omega t + \phi)$$
$$\dot{x} = -A\omega \quad (\omega t + \phi)$$
$$\ddot{x} = -A\omega^2\cos(\omega t + \phi)$$

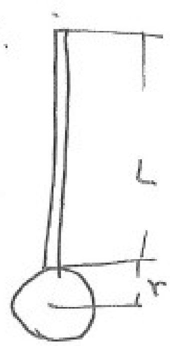
$$kA\cos(\omega t+\phi) + \frac{3}{2}m\left(-A\omega^2\cos(\omega t+\phi)\right) = 0$$

$$A\cos(\omega t+\phi)\left(k - \frac{3}{2}m\omega^2\right) = 0$$

$$\therefore \quad \omega = \sqrt{\frac{2k}{3M}}$$

$$\therefore \quad P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2k}}$$

102.



$r = 0.15\,m$

$P = 2.0\,sec$

$g = 9.800\,m/sec^2$

$$I = (L+r)^2 m + \frac{1}{2}mr^2$$

$$\omega = \sqrt{\frac{mgd}{I}} \quad \& \quad P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

$$\therefore \quad P = 2\,sec = 2\pi\sqrt{\frac{m(L+r)^2 + \frac{1}{2}mr^2}{mg(L+r)}}$$

$$\vdots$$

$$\pi L^2 + (2\pi r - g)L + \left(\frac{3}{3}\pi^2 r^2 - gr\right) = 0$$

$$\therefore \quad L = 0.831485381\,m$$

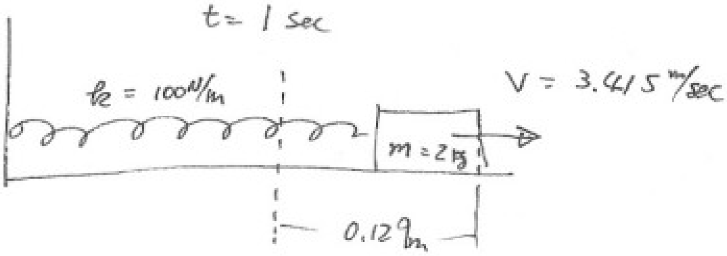
Ch. 15 additional problems:

#1 A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. What t = 1.00 sec, the position and velocity of the block are x = 0.129 m and v = 3.415 m/sec. (a) What is the amplitude of the oscillations? What were the (b) position and (c) velocity of the block at t = 0 sec?

#2 A massless spring hangs from the ceiling with a small object attached to its lower end. The object is initially held at rest in a position y_i and oscillates up and down, with its lowest position being 10 cm below y_i. (a) What is the frequency of the oscillation? (b) What is the speed of the object when it is 8.0 cm below the initial position? (c) An object of mass 300 g is attached to the first object, after which the system oscillates with half the original frequency. What is the mass of the first object? (d) Relative to y_i, where is the new equilibrium (rest) position with both objects attached to the spring?

#3 Suppose that the two spring in fig 15-30 have different spring constant k_1 and k_2. Show that the frequency f of oscillation of the block is then given by $\sqrt{\nu_1^2 + \nu_2^2}$

#1



$$\left.\begin{aligned} x &= A\cos(\omega t+\phi) \\ \dot{x} &= -A\omega \sin(\omega t+\phi) \\ \ddot{x} &= -A\omega^2 \cos(\omega t+\phi) \end{aligned}\right\} \quad t = 1\text{ sec} \rightarrow \left\{\begin{aligned} x &= A\cos(\omega+\phi) = 0.129 \quad -① \\ \dot{x} &= -A\omega\sin(\omega+\phi) = 3.415 \quad -② \\ \ddot{x} &= -A\omega^2\cos(\omega+\phi) \end{aligned}\right.$$

So,

Eqn. ①. $\quad 0.129 = A\cos\left(\sqrt{\frac{k}{m}} + \phi\right) \qquad \left(\omega = \sqrt{\frac{k}{m}}\right)$

Eqn ② $\quad 3.415 = -A\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}} + \phi\right) \rightarrow 3.415\sqrt{\frac{m}{k}} = -A\sin\left(\sqrt{\frac{k}{m}} + \phi\right) \quad -②'$

$①^2 + ②^2$

a) $A^2\cos^2\left(\sqrt{\frac{k}{m}} + \phi\right) + A^2\sin^2\left(\sqrt{\frac{k}{m}} + \phi\right) = (0.129)^2 + \left(3.415\sqrt{\frac{m}{k}}\right)^2$

$$A^2(\underbrace{\cos^2 + \sin^2}_{1}) = (0.129)^2 + \left(3.415\sqrt{\frac{2}{100}}\right)^2$$

$$A^2 = (0.129)^2 + \left(3.415\sqrt{\frac{2}{100}}\right)^2$$

$$\therefore\ A = 0.4998854869\text{ m}$$

(b) First we need to calculate ϕ

$$\tan(\omega t+\phi) = \frac{\sin(\omega+\phi)}{\cos(\omega+\phi)} = \frac{②'}{①'} = \frac{-3.415\sqrt{\frac{m}{k}}}{0.129}$$

[to calculate an angle, it is always a good idea to use $\tan^{-1}$ because it tells us which quad. we are looking at. For example, in this case, if only eqn. ① was used, we would think the ans. was +. However, by looking at both sin & cos., we know the ans. is in the 4th quad.]

$$\sqrt{\frac{k}{m}} + \phi = \tan^{-1}\frac{-3.415\sqrt{\frac{2}{100}}}{0.129}$$

$$= -1.309783607 \text{ rad}$$

$$\therefore \phi = -8.380851419 \text{ rad}$$

$$x = A\cos(\omega t+\phi)$$

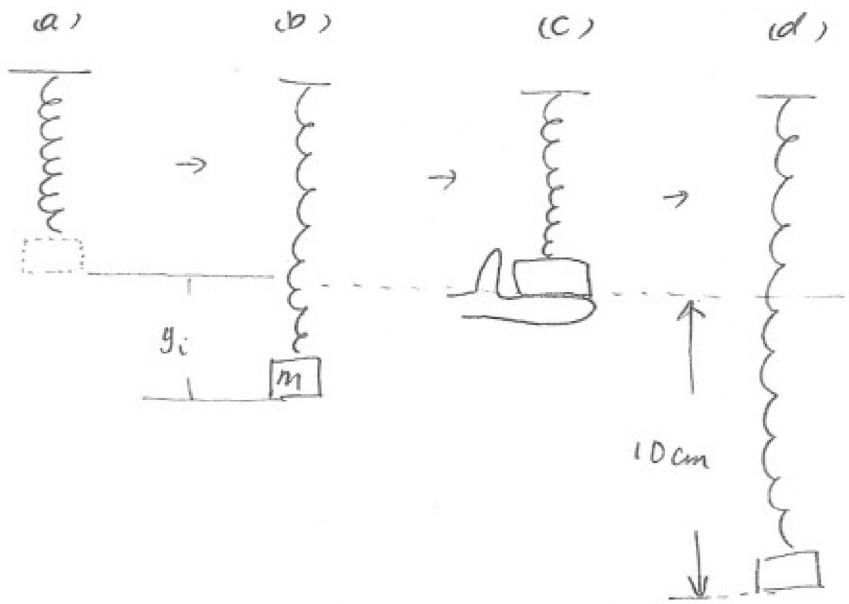
$$= 0.498854869\cos(\cancel{\omega t}^{0} - 8.380851419)$$

$$= \underline{\underline{-0.251357467 \text{ m}}}$$

(c) $\dot{x} = -A\sqrt{\frac{k}{m}}\sin(\cancel{\omega t}^{0} - 8.380851419)$

$$= \underline{\underline{3.055363503 \text{ m/sec}}}$$

#2



(a) In the text book, they use f as frequency, but I will use ν (nu) for frequency.

$$m = \frac{1}{4}(m + 0.3)$$

$$m - \frac{1}{4}m = \frac{0.3}{4}$$

$$\frac{3}{4}m = \frac{0.3}{4}$$

$$m = \frac{0.3}{3} = \underline{\underline{0.1 \text{ kg}}}$$

(d) $$F = -kx = -mg$$

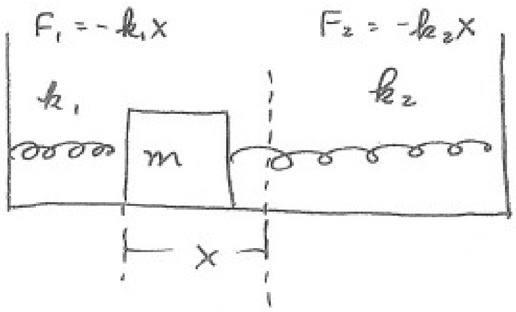
$$k = \frac{mg}{x} = \frac{0.1 \cdot 9.81}{0.05} = \frac{98.1}{5}$$

$$F_{new} = -kX_{new} = -m_{new}\, g$$

$$X = \frac{(0.1 + 0.3)\, 9.81}{\frac{98.1}{5}}$$

$$= \frac{5 \cdot 0.4 \cdot 9.81}{98.1} = \underline{\underline{0.2 \text{ m}}}$$

11 # 3



$$F_{net} = F_1 + F_2 = -(k_1 x - k_2 x) = -(k_1 + k_2)x$$

$$ma = -(k_1 + k_2)x$$

$$m\ddot{x} + (k_1 + k_2)x = 0$$

Let

$$x = A\cos(\omega t + \phi)$$

$$\ddot{x} = -A\omega^2\cos(\omega t + \phi)$$

$$m\left(-A\omega^2\cos(\omega t + \phi)\right) + (k_1 + k_2)A\cos(\omega t + \phi) = 0$$

$$A\cos(\omega t + \phi)\left(-m\omega^2 + (k_1 + k_2)\right) = 0$$

$$m\omega^2 = k_1 + k_2$$

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

How can we find ω (thus frequency) w/o any k a m?
The spring stretches when a mass is suspended from it (fig.(b))
because there is a force (weight of m in this case) acting
on the spring.

$$F = -k y_i = -mg \qquad y_i \text{ is a half of } \triangle \text{ displacement } = 0.05\,m$$

$$\Rightarrow \frac{k}{m} = \frac{g}{y_i}$$

$$\underbrace{\omega = \sqrt{\frac{k}{m}}} = \sqrt{\frac{g}{y_i}} = \sqrt{\frac{9.81}{0.05}} = \underline{\underline{2.229365734\ Hz}}$$

You should be able to derive this.

(b)
$$x = 0.05 \cos(\omega t + \phi) \text{ ——— ①}$$
$$\dot{x} = -0.05\,\omega \sin(\omega t + \phi)$$
$$= -0.05\sqrt{\frac{9.81}{0.05}} \sin(\omega t + \phi) \text{ ——— ②}$$

eqn ①
$$x = 8-5\text{ cm} = 0.03\,m = 0.05 \cos(\omega t + \phi)$$
$$\therefore \frac{0.03}{0.05} = \cos(\omega t + \phi)$$
$$\therefore \cos^{-1}\frac{0.03}{0.05} = (\omega t + \phi) \text{ ——— ①}'$$

② ⟵ ①′

$$|\dot{x}| = \left| -0.05\sqrt{\frac{9.81}{0.05}} \sin\left(\cos^{-1}\frac{0.03}{0.05}\right)\right| = \underline{\underline{0.5602856444\ m/sec}}$$

(c)
$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{y_i}} = \sqrt{\frac{9.81}{0.05}} \Rightarrow \nu_0 = \frac{\omega_0}{2\pi}$$

$$\omega_0 = \sqrt{\frac{k}{m+0.3}} \Longrightarrow \nu = \frac{\omega}{2\pi}$$

$$\frac{\nu}{\nu_0} = \frac{\frac{\omega}{2\pi}}{\frac{\omega_0}{2\pi}} = \frac{\omega}{\omega_0} = \frac{\sqrt{\frac{k}{m+0.3}}}{\sqrt{\frac{k}{m}}} = \sqrt{\frac{m}{m+0.3}} = \frac{1}{2}$$

$$\frac{m}{m+0.3} = \left(\frac{1}{2}\right)^2$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \quad (\text{if } k_1 = k_2)$$

modefying $\omega = \sqrt{\frac{k}{m}}$

$$\omega_1 = \sqrt{\frac{k_1}{m}} \quad \& \quad \omega_2 = \sqrt{\frac{k_2}{m}}$$

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad \& \quad \nu_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$$

$$\therefore \frac{k_1}{m} = 4\pi^2\nu_1^2 \quad \& \quad \frac{k_2}{m} = 4\pi\nu_2^2$$

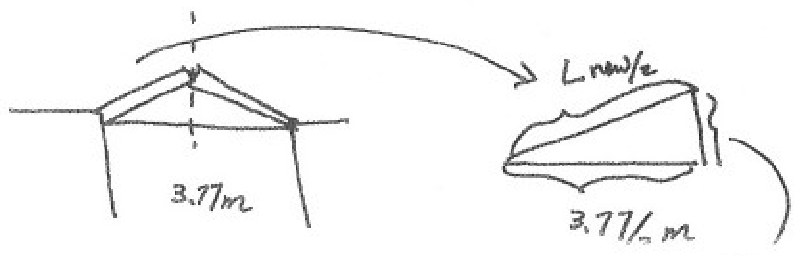
$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{k_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{4\pi^2\nu_1^2 + 4\pi\nu_2^2}$$

$$= \frac{1}{2\pi}\sqrt{4\pi^2(\nu_1^2 + \nu_2^2)} = \underline{\underline{\sqrt{\nu_1^2 + \nu_2^2}}}$$

ch 18. #21, 30, 37, 3, 41, 42, 43, 44, 45, 48, 49, 54, 59

#21



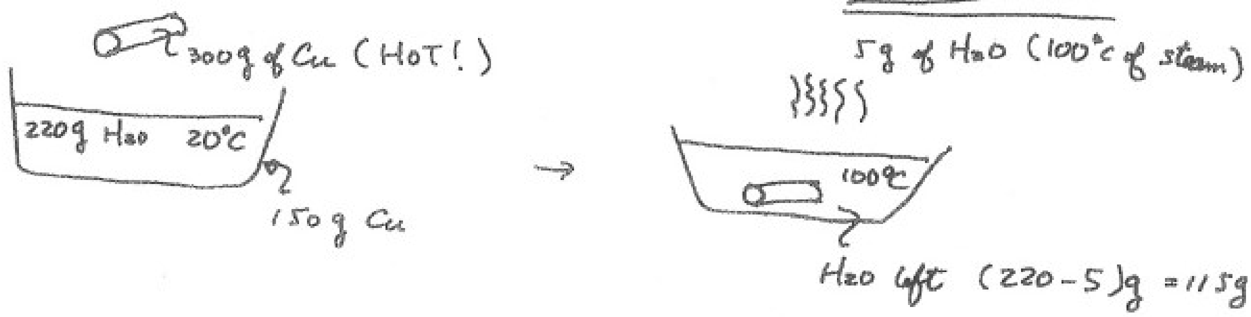
$\Delta T = 32°C$
$\alpha = 25 \times 10^{-6}/°C$

$L_{new} = L + L\alpha\Delta T$
$= 3.773016\,m$

$$\left[\left(\frac{3.773016}{2}\right)^2 - \left(\frac{3.77}{2}\right)^2\right]^{1/2} = 0.07541507 8\,m$$

$= \underline{\underline{7.5415078\,cm}}$

#30



(a)

ΔQ_{H_2O} = E (220g of water brought from 20°C → 100°C) +
E (5g of water brought from 100°C of water to 100°c of steam)

$= (SHC \cdot mass \cdot \Delta T)_{H_2O} + L_V \cdot mass$ $\qquad L_V = 2256\ KJ/kg$

$= 1\ cal/g/°C \cdot 220g \cdot (100-20)°C + 538.939\ldots cal/g \cdot 5g$ $\qquad = 2256\ J/g$

$= 17600\ cal + 2694.696608\ cal$ $\qquad = 538.9393215\ cal/g$

$= \underline{\underline{20294.69611\ cal}}$

(b)

$\Delta Q_{Cu} = SHC_{Cu} \cdot mass \cdot \Delta T$

$= 0.0923\ cal/g/°C \cdot 150g \cdot (100-20)°C$

$= \underline{\underline{1107.6\ cal}}$

(c)

$+\Delta Q$ to H_2O & Cu bowl $- Q$ from the Cu cylinder $= 0$.

$(20294.69611\ cal + 1107.6\ cal) - 0.0923\ cal/g/°C \cdot 300g \cdot \Delta T$

$$\Delta T = \frac{21402.29661\ cal}{0.0923\ cal/g/°C \cdot 300g} = 772.9251213°C \quad (\text{from } T_f = 100°C)$$

$\therefore T_i = \Delta T + T_f = 872.9251213°C$

#37

(a) First we need to check 500 g of 90°C water has enough energy to melt 500 g of 0°C ice.

$$\begin{bmatrix} SHC_{H_2O} = 4.190 \text{ J/g/°C} \\ L_F = 333 \text{ J/g} \end{bmatrix}$$

Energy above 0°C (for 90°C water)

$4.190 \text{ J/g/°C} \cdot 500\text{g} \cdot 90°\text{C}$

$= 1.8855 \times 10^5 \text{ J}$

Energy needed to melt 0°C ice

$L_F \cdot \text{mass}$

$333 \text{ J/g} \cdot 500\text{g}$

$= 1.665 \times 10^5 \text{ J}$

As you can see, the 90°C water has barely enough to melt all ice. The left over energy is shared by (500g + 500g) of water at the same temperature.

$$\therefore \quad (1.8855 \times 10^5 \text{J} - 1.665 \times 10^5 \text{J}) = 4.190 \text{ J/g/°C} \cdot (500\text{g} + 500\text{g})\, T_f$$

$$T_f = \frac{(1.8855 \times 10^5 \text{J} - 1.665 \times 10^5 \text{J})}{4.190 \text{ J/g/°C} \cdot 1000\text{g}} = \underline{\underline{5.26\dot{2}52983\dot{3}\,°C}}$$

(b)

Energy above 0°C (for 70°C water)

$4.190 \text{ J/g/°C} \cdot 500\text{g} \cdot 70°\text{C}$

$= 1.46650 \times 10^7 \text{ J}$ (This is not enough to melt all the ice!)

How many grams of ice can this energy melt?

$1.46650 \times 10^7 \text{J} \div 333 \text{ J/g} = \underbrace{440.3903904 \text{ g}}$

So, unmelted ice is $500\text{g} - \swarrow = \underline{\underline{5\dot{9}.\dot{6}\dot{9} \text{ g}}}$

#39

($SHC_{ice} = 2.220 \text{ J/g/°C}$)

Energy of H_2O above 0°C

$4.190 \text{ J/g/°C} \cdot 200\text{g} \cdot 25°\text{C}$

$= 20950 \text{ J}$

Energy of ice <u>below</u> 0°C water level

$2.220 \text{ J/g/°C} \cdot (50\text{g} \times 2) + 333 \text{ J/g/°C} \cdot (50\text{g} \times 2)$

$= 3330 \text{ J} + 33300 \text{ J}$

$= 36630 \text{ J}$

(a) $E_T = -15680 \text{ J}$ (15680 J lower than 0°C water level)

$15680 \text{ J} \div 333 \text{ J/g} = 47.\dot{0}8\dot{7} \text{ g}$ (to freeze ice)

$\Rightarrow$ $\underline{\underline{47.\dot{0}8\dot{7} \text{ g of 0°C ice \& } 252.\dot{9}1\dot{2} \text{ g of 0°C water}}}$

(b) 50 g of ice instead of 100 g.

Energy of ice below 0°C water level

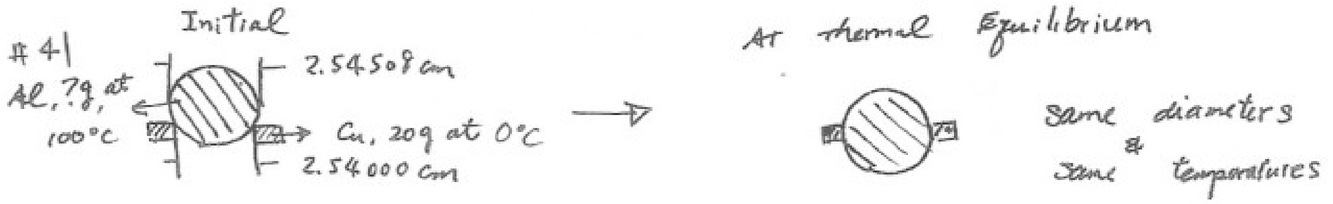
$2.22\ J/g/°C \cdot 50g + 333\ J/g/°C \cdot 50g$

$= 1665\ J + 16650\ J = 18315\ J$

$E_T = 20950J - 18315\ J = 2605\ J$ (This is shared by (200 + 50) g water)

$2605\ J \div (4.190\ J/g/°C \cdot 250g) = \underline{\underline{2.544663415°C}}$

\#41



. Thermal Expansion

Dia. of Cu = Dia. of Al (at thermal equilibrium)

$L_{cu} + L_{cu}\alpha_{cu}\Delta T = L_{Al} + L_{Al}\alpha_{Al}\Delta T$

$$\left[\begin{array}{l} \alpha_{cu} = 17\times10^{-6}/°C \\ \alpha_{Al} = 23\times10^{-6}/°C \\ SHC_{cu} = 0.0923\ cal/g/°C \\ SHC_{Al} = 0.215\ cal/g/°C \end{array}\right]$$

$2.54000\,cm + 2.54\,cm \cdot 17\times10^{-6}/°C\,(T_f - \cancel{T_{i,cu}}) = 2.54508\,cm + 2.54508\,cm \cdot 23\times10^{-6}/°C\,(T_f - \overset{100°C}{T_i})$

$2.54 + 2.54 \cdot 17\times10^{-6}/°C \cdot T_f = 2.54508 + 2.54508 \cdot 23\times10^{-6}\,T_f - 254.508 \cdot 23\times10^{-6}/°C$

Solve for T_f

$$T_f = \frac{2.54 - 2.54508 + 254.508 \cdot 23\times10^{-6}}{2.54508 \cdot 23\times10^{-6} - 2.54 \cdot 17\times10^{-6}} = \underline{50.81397787\ °C}$$

cons. of E

$$\overset{E_i}{SHC_{cu} \cdot m_{cu} \cdot \overset{0}{\cancel{T_{i,cu}}} + SHC_{Al} \cdot m_{Al} \cdot \overset{100}{T_{i,Al}}} = \overset{E_f}{(SHC_{cu} \cdot m_{cu} + SHC_{Al} \cdot m_{Al})\,T_f}$$

$100\ SHC_{Al} \cdot m_{Al} - SHC_{Al} \cdot m_{Al}\,T_f = SHC_{cu} \cdot m_{cu} \cdot T_f$

$(100 - T_f)\ SHC_{Al} \cdot m_{Al} = SHC_{cu} \cdot m_{cu} \cdot T_f$

$$\therefore m_{Al} = \frac{SHC_{cu} \cdot m_{cu} \cdot T_f}{(100 - T_f)\,SHC_{Al}}$$

$$= \frac{0.0923 \cdot 20 \cdot 50.81397787}{(100 - 50.81\ldots)\ 0.215}$$

$= 8.870226918\ g$

#42 (a)

	Q	W	ΔE
A → B	+	+	+
B → C	+	0	+
C → A	—	—	—

W is + because it moved to +
Q is + because $\Delta E = Q - W$
W is 0 because it went up straight (No vol change)
E is + because $\Delta E = Q$
W is — because it moved to —
ΔE is — because it has to come back to the same state
Q is — because $\Delta E = Q - W$.

(b)

A → B	40 J
B → C	0
C → A	−60 J
Net	−20 J

(outside did 20 J of work to the system)

#43

Calculate the area enclosed by path A, B or C

$$\left(W = \int_{Vol_i}^{Vol_f} Pr \cdot d(vol) \right)$$

A: $40\,(4-1) = \underline{120\text{ J}}$

B: $120\text{ J} - 45\text{ J} = \underline{75\text{ J}}$

C: $10\,(4-1) = \underline{30\text{ J}}$

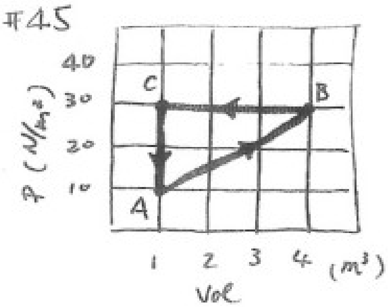
#44

(a) $W = \underline{-200\text{ J}}$

(b) $Q = -70\text{ cal} = \underline{-293.02\text{ J}}$

(c) $\Delta E_{int} = Q - W = \underline{-93.02\text{ J}}$

#45



As you can see $\overline{AB}$ is the time when the positive work is done to the system, and $\overline{BC}$ is when the system is doing the work (Energy is going out). No work is done during $\overline{CA}$. Also notice that this system is releasing more energy to outside ($\overline{BC}$ phase) than taking ($\overline{AB}$ phase). The net change is the area of the triangle ABC.

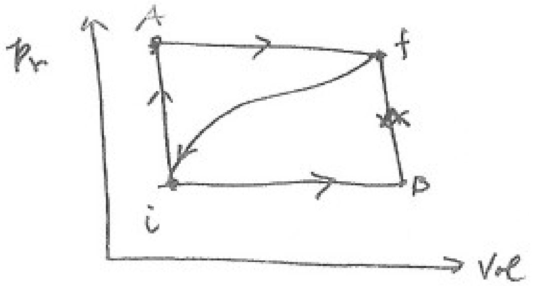
$\therefore \frac{1}{2}(30-10)(4-1)$ J or −30 J of work is done to the system or 30 J of work is done to the outside by the system.

#48 $Q_{AB} + Q_{BC} + Q_{CA} = W$

$\therefore Q_{CA} = W - Q_{AB} - Q_{BC}$

$= 15J - 20J - 0J = \underline{\underline{-5J}}$ (the system emits 5J to outside)

#49



iaf $Q = 50$ cal, $W = 20$ cal

ibf $Q = 36$ cal, $W = ?$

(a) No matter which path it takes ΔE_{int} from 'i' & 'f' should be the same.

using iaf: $\Delta E_{int} = Q - W = 50cal - 20cal = 30cal.$

so, using ibf, $\Delta E_{int} = 30cal.$

$\therefore \Delta E_{int} = 30cal = Q - W = 36cal - W$

$\therefore \underline{\underline{W = 6\ cal}}$

(b) Going back to 'i' from 'f', $\Delta E = -30$ cal (reverse of (a))

$-30cal = Q - (-13\ cal)$

$\therefore \underline{\underline{Q = -43cal}}$

(c) $E_{ib} = 22cal$

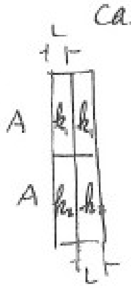
$\Delta E = E_{ib} + E_{bf}$

$30\ cal = 22cal + E_{bf}$

$\therefore \underline{\underline{E_{bf} = 18cal = Q}}$ because $E_{bf} = Q_{bf} - W_{bf}$ & $W_{bf} = 0$ (No vol change)

\# 54

case (a)

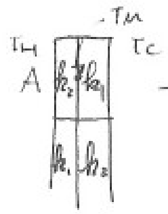


$$\rightarrow P_1 = k_1 \frac{A(T_H - T_C)}{2L}$$

$$\rightarrow P_2 = k_2 \frac{A(T_H - T_C)}{2L}$$

$$P_T = P_1 + P_2 = \underline{(k_1 + k_2)\frac{A(T_H - T_C)}{2L}} \quad \text{— ①}$$

case (b)



$$\rightarrow \quad P = k_2 \frac{A(T_H - T_M)}{L} = k_1 \frac{A(T_M - T_C)}{L}$$

Solve for T_M

$$k_2(T_H - T_M) = k_1(T_M - T_C)$$

$$T_M = \frac{k_2 T_H + k_1 T_C}{k_1 + k_2} \Rightarrow \text{sub. back to the 1st eqn.}$$

$$\therefore P = k_2 \frac{A\left(T_H - \frac{k_2 T_H + k_1 T_C}{k_1 + k_2}\right)}{L}$$

$$= \frac{k_2 A}{L}\left(T_H(k_1 + k_2) - (k_2 T_H + k_1 T_C)\right)$$

$$= \underline{\frac{A}{L}\,\frac{k_1 k_2 (T_H - T_C)}{k_1 + k_2}} \Rightarrow P_T = 2 \times P = \underline{\frac{2A}{L}\,\frac{k_1 k_2 (T_H - T_C)}{k_1 + k_2}} \quad \text{—②}$$

①

$$\frac{(k_1 + k_2)}{2} \cdot \frac{A(T_H - T_C)}{L}$$

②

$$\frac{2 k_1 k_2}{(k_1 + k_2)} \cdot \frac{A(T_H - T_C)}{L}$$

So which is bigger?

$$\left(\frac{k_1 + k_2}{2}\right) - \left(\frac{2k_1 k_2}{k_1 + k_2}\right) \overset{?}{\gtrless} 0$$

$$\frac{k_1 + k_2}{2} \overset{?}{\gtrless} \frac{2k_1 k_2}{k_1 + k_2}$$

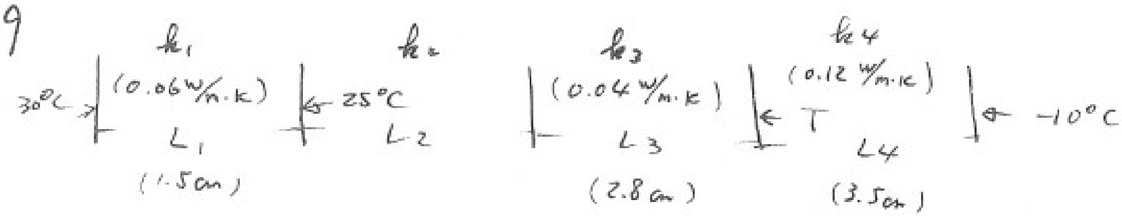
$$(k_1 + k_2)^2 - 4 k_1 k_2$$

$$= k_1^2 + k_2^2 - 2k_1 k_2$$

$$= (k_1 - k_2)^2 > 0 \quad (k_1 \neq k_2)$$

$$\therefore \underline{P_{case\,(a)} > P_{case\,(b)}}$$

#59



Key sentence: Energy transfer through the wall is steady

$$\therefore \quad P_{cond} = k_1 A \frac{(T_H - T_{C,1})}{L_1} = k_4 A \frac{(T - T_{C,4})}{L_4}$$

$$k_1 \frac{(T_H - T_{C,1})}{L_1} = k_4 \frac{(T - T_{C,4})}{L_4}$$

$$\therefore \quad T = T_C + \frac{k_1 L_4}{k_4 L_1} (T_H - T_{C,1})$$

Chapter 18 additional problems $\quad = -10°C + \frac{0.06 \cdot 3.5\,cm}{0.12 \cdot 1.5\,cm}(30°C - 25°C) = \underline{-4.16°C}$

#1 The Pyrex glass mirror in the telescope at the Mr. Palomar Observatory has a diameter of 200 inches. The temperature ranges from –10 ^{0}C to 50 ^{0}C on Mt. Palomar. In micrometers, what is the maximum change in the diameter of the mirror, assuming that the glass can freely expand and contract?

#2 The area A of a rectangular plate is "ab". Its coefficient of linear expansion is α. After a temperature rise ΔT, side a is longer by Δa and side b by Δb. Show that if the small quantity $(\Delta a \Delta b)/ab$ is neglected, then $\Delta A = 2\alpha A \Delta T$.

#3 A pendulum clock with a pendulum made of brass is designed to keep accurate time at 20 ^{0}C. If the clock operates at 0.0 ^{0}C, what is the magnitude of its error, in seconds per hour, and does the clock run fast or slow?

#4 How much water remains unfrozen after 50.2 kJ is transferred as heat from 260 g of liquid water initially at its freezing point?

#5 An energetic athlete can use up all the energy from a diet of 4000 Cal/day. If he were to use up this energy at a steady rate, how would his rate of energy use compare with the power of a 100 W bulb?

#6 How many grams of butter, which has a usable energy content of 6.0 Cal/g, would be equivalent to the change in gravitational potential energy of a 73.0 kg man who ascends from sea level to the top of Mt. Everest, at elevation 8.84 km? Assume that the average value of g is 9.80 m/sec^2.

#7 Suppose that a man holding a container, and there is water inside. If he is making 30 shakes each minute and the water 30 cm each shake, how long must he shake the container to change water temperature from 15 ^{0}C to 100 ^{0}C? Neglect any loss of thermal energy by the container.

#8 In a solar water heater, energy from the sun is gathered by water that circulates through tubes in a rooftop collector. The solar radiation energy the collector through a transparent cover and warms the water in the tubes; this water is pumped into a holding tank. Assume that the efficiency of the overall system is 20%. What collector area is necessary to raise the temperature of 200 L of water in the tank from 20 ^{0}C to 40 ^{0}C in 1.0 hour when the intensity of incident sunlight is 700 W/m^2?

#1 $200\text{ in} = 508\text{ cm}$

$\alpha_{pyrex} = 3.2\times10^{-6}/°C$

$\Delta L = L\alpha\Delta T$

$= (508\text{ cm})(3.2\times10^{-6}/°C)(50-(-10))°C$

$= 9.7536\times10^{-4}\text{ m}$

$= \underline{\underline{975.36\ \mu m}}$

#2

$A_{before} = a\cdot b$

$A_{after} = (a+\Delta a)(b+\Delta b)$

$= ab + a\Delta b + \Delta a b + \cancel{\Delta a\Delta b}^{\to 0}$

$= ab + a(b\alpha\Delta T) + (a\alpha\Delta T)b$

$= ab + ab\,\alpha\Delta T + ab\alpha\Delta T$

$= ab + 2\underbrace{ab}\alpha\Delta T$

$= A_{before} + 2A_{before}\alpha\Delta T$

$= \underline{\underline{A + A(2\alpha)\Delta T}}$

#3 simple pendulum : $\omega = \sqrt{\frac{g}{L}}$ (You should be able to prove that.)

$\therefore\ P_{20°C} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \frac{2\pi}{\sqrt{g}}\sqrt{L}$

$P_{0°C} = 2\pi\sqrt{\frac{L+L\alpha\Delta T}{g}}$ $\left(\begin{array}{l}\alpha_{brass} = 19\times10^{-6}/°C\\ \Delta T = 0°C - 20°C = -20°C\end{array}\right)$

$= 2\pi\sqrt{\frac{0.99962L}{g}} = \frac{2\pi}{\sqrt{g}}\sqrt{0.99962L}$

$\frac{P_{0°C}}{P_{20°C}} = \frac{\frac{2\pi}{g}\sqrt{0.99962L}}{\frac{2\pi}{g}\sqrt{L}} = \sqrt{0.99962} = 0.99980998$

$\Delta T/_{hr} = \frac{3600\text{ sec} - 3600\text{ sec}(0.99980998)}{hr} = \underline{\underline{0.684065\text{ sec}/_{hr}\text{ faster}}}$

#4

$Q = 50.2\ kJ\ (50200J)$

$L_F = 333\ kJ/kg = 333\ J/g$

$50200\ J \div 333\ J/g = 150.75\ g$ of 0 °C of ice can be melted to 150.75 g of 0 °C of water.

$\therefore\ 260\ g - 150.75\ g = \underline{\underline{109.2492493\ g}}$ of ice is left

#5

$4000\ cal/day = 4\times10^6\ cal/day = 1.6744\times10^7\ J/day$ ($1\ cal = 4.186\ J$)

$P = \frac{W}{t} = \frac{1.6744\times10^7 J}{86400\ sec\ (=1\ day)} = \underline{\underline{193.7962963\ Watts}}$

(~ 2 × 100 watt light bulbs)

#6

$mg\Delta h = 73kg\ (9.80\ m/sec^2)\ 8840m = 6.324136\times10^6 J$

$6.324136\times10^6 J \div (6000\ cal/g \cdot 4.186\ J/cal) = \underline{\underline{251.7971064\ g}}$ of butter

(~ 2 sticks of butter: No wonder we gain weight!)

#7

Every shake $\Rightarrow dQ = d(mg\Delta h)$

$Q_{needed} = m \cdot SHC \cdot \Delta T$

$= m \cdot 4190\ J/kg/°C \cdot (100°C - 15°C)$

$= 3.5615\times10^5\ m\ J/kg$

$\#\ shakes\ Needed = \frac{Q_{needed}}{dQ} = \frac{3.5615\times10^5\ \cancel{m}\ J/kg}{\cancel{m}\cdot 9.81\cdot 0.3} = 121015.9701$ shakes

$t = \frac{\#\ shakes\ needed}{30\ shakes/\cancel{sec}\ min} \cdot \frac{1\ hr}{60\ min} \cdot \frac{1\ day}{24\ hrs} = \underline{\underline{2.801295604\ days}}$

(Good Luck!)

#8

$200L$ of $H_2O = 200{,}000g$ of H_2O

[$1L = 1000ml$
$= 1000\ cm^3$
$= 1000\ g$ since $\rho_{H_2O} = 1\ g/cm^3$]

Also

$700\ W/m^2 \times 0.2 = 140\ \frac{J/sec}{m^2} \Rightarrow 140\frac{J/sec}{m^2} \cdot \frac{60\ sec}{1\ min} \cdot \frac{60\ min}{1\ hr} = 50400\ \frac{J/hr}{m^2}$ —— ①

Energy needed to raise 200 L of H_2O from 20°C to 40°C (in 1 hr)

$SHC \cdot mass \cdot \Delta T$

$4.190\ J/g/°C \cdot 200000 \cdot (40°C - 20°C) = 1.676\times10^7 J$ —— ②

② ÷ ①

$1.676\times10^7\ J/hr \div 50400\ \frac{J/hr}{m^2} = 33.25396825\ m^2$